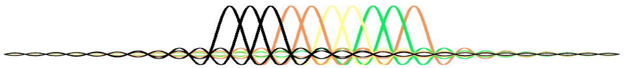

ELEC 450 & ELEC 550, Introduction to Mobile Broadband

Lecture 6: OFDM

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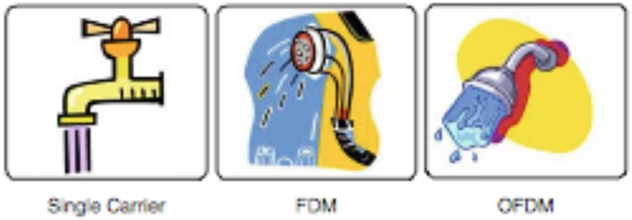
Fall, 2013
Koc University

courses.ku.edu.tr/elec450



Outline

- What is OFDM?
- Coding
- Synchronization
- Equalization
- Peak to Average Power Ratio
- Ex: IEEE 802.11a

What is OFDM?

- Multi-carrier modulation/multiplexing technique
- Available bandwidth is divided into several *subchannel*
- Data is serial-to-parallel converted
- Symbols are transmitted on different *subcarriers*

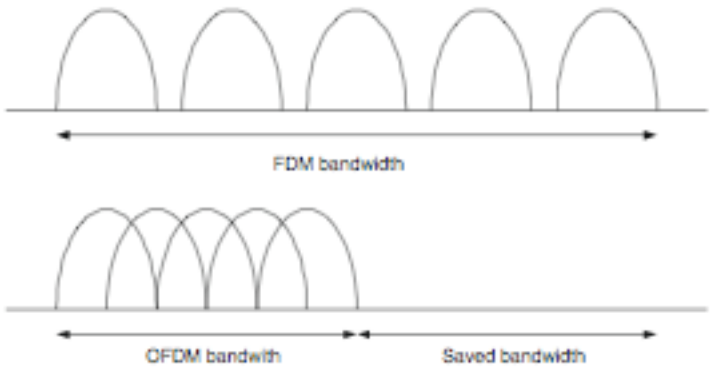
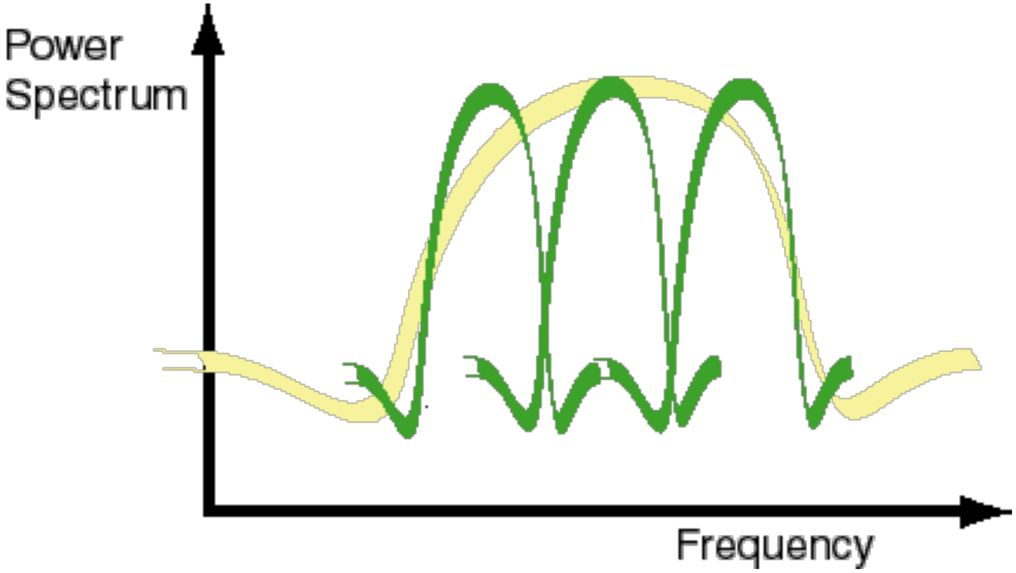


Fig. 4.1 Comparison of OFDM and FDM

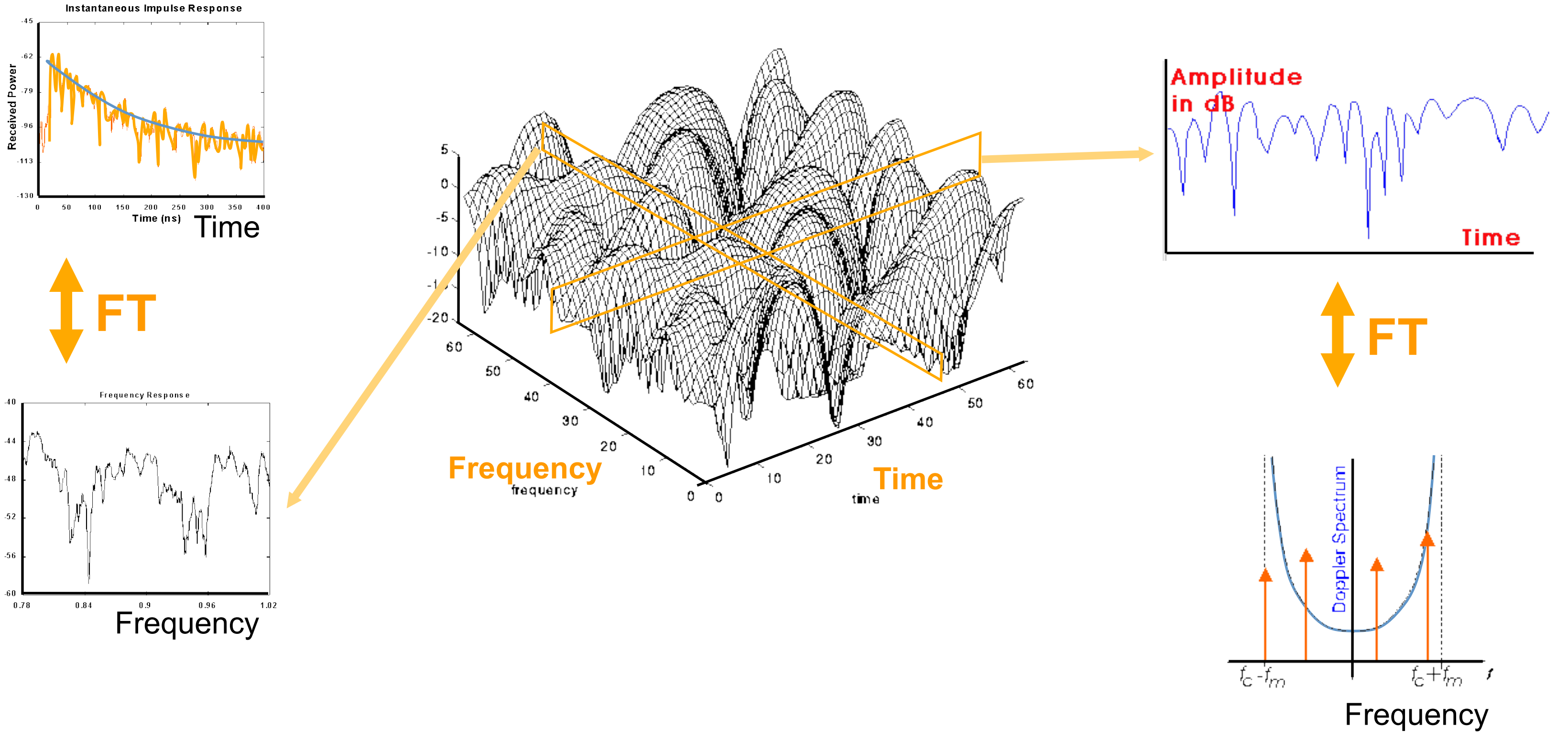
OFDM



- Spectra overlap, but signals are orthogonal.
- Example: Rectangular waveform -> Sinc spectrum

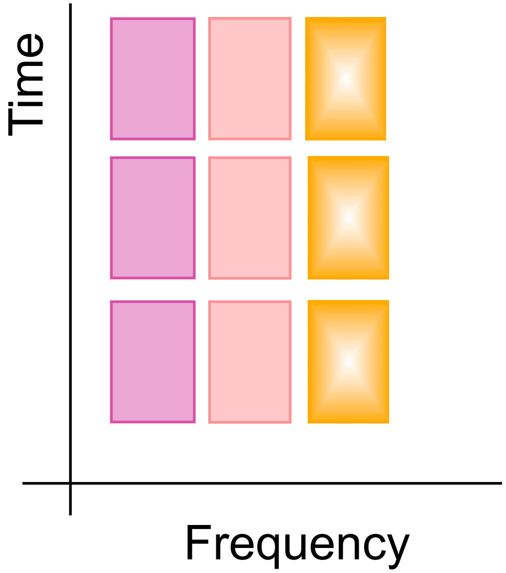
Wireless Multipath Channel

- Delay spread
- Doppler spread

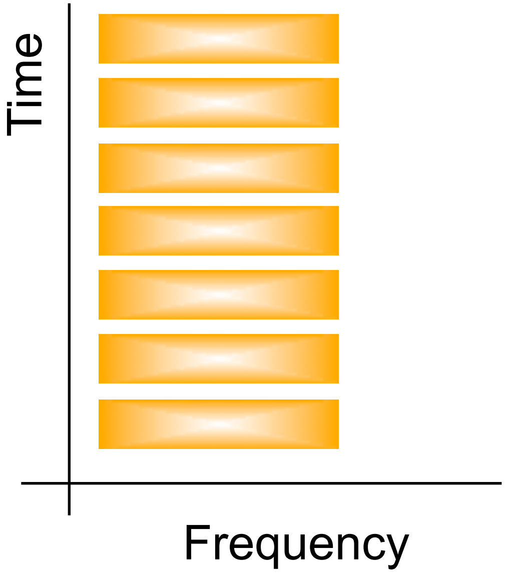


Effects of Multipath Delay and Doppler

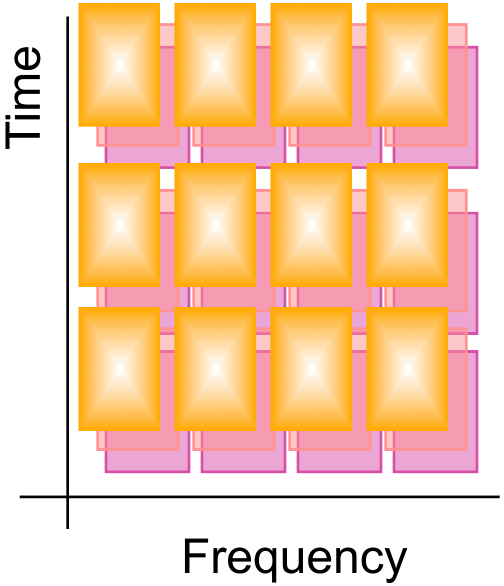
Narrowband



Wideband
QAM



OFDM



OFDM

- Advantages
  - Spectral efficiency
  - Simple implementation
  - Tolerant to ISI
- Disadvantages
  - BW loss due guard time
  - Prone to frequency and phase offset errors
  - Peakto average power-problem

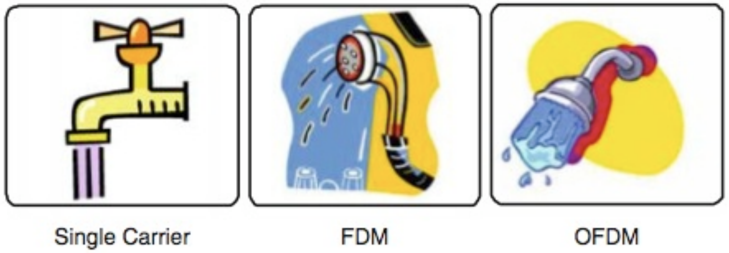


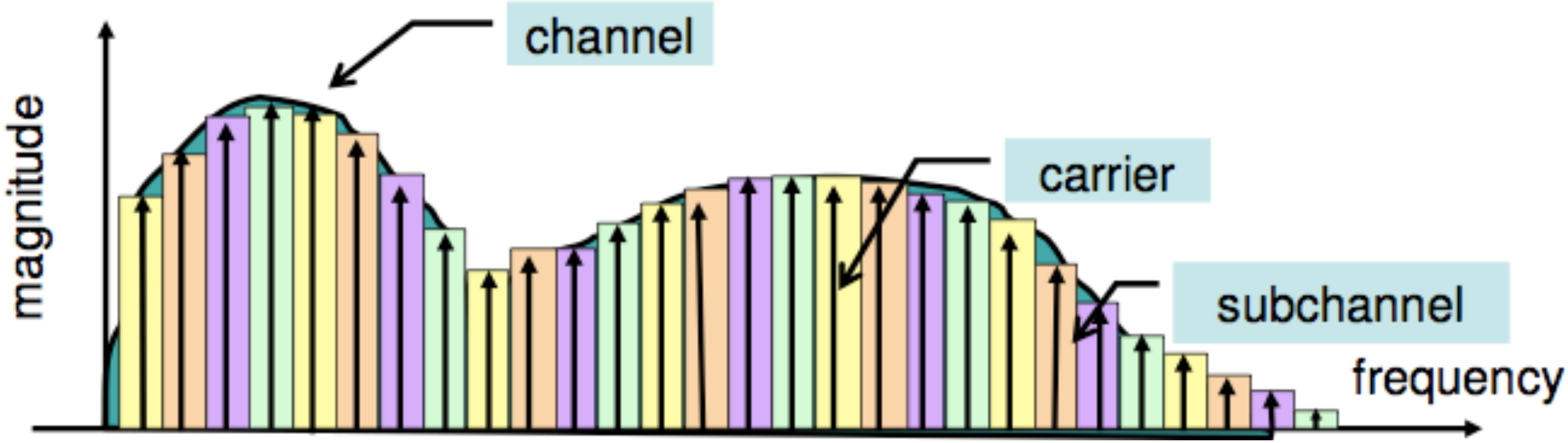
Fig. 4.2 Comparison of OFDM over FDM and single-carrier systems. OFDM and FDM are resilient to interference, since flow of water can be easily stopped in single-carrier systems. OFDM is more spectral efficient than FDM, since it utilizes the surface effectively with adjacent tiny streams

Applications

- Fixed / Wireline:
  - ADSL Asymmetric Digital Subscriber Line
- Mobile / Radio:
  - Digital Audio Broadcasting (DAB)
  - Digital Video Broadcasting - Terrestrial (DVB-T)
  - Hiperlan II
  - Wireless 1394
  - Wireless LAN: IEEE 802.11a/n
  - 4G: OFDMA based WiMAX, LTE

What is subchannel and subcarrier?

- Frequency-selective channel is divided into flat fading subchannels
- Fast serial data stream is transformed into slow parallel data streams – Longer symbol durations



Subchannels

- OFDM spectrum subchannel spacings are selected so, that they are mathematically orthogonal to each other

- Subchannels overlap on each other – Sinc-shaped spectra

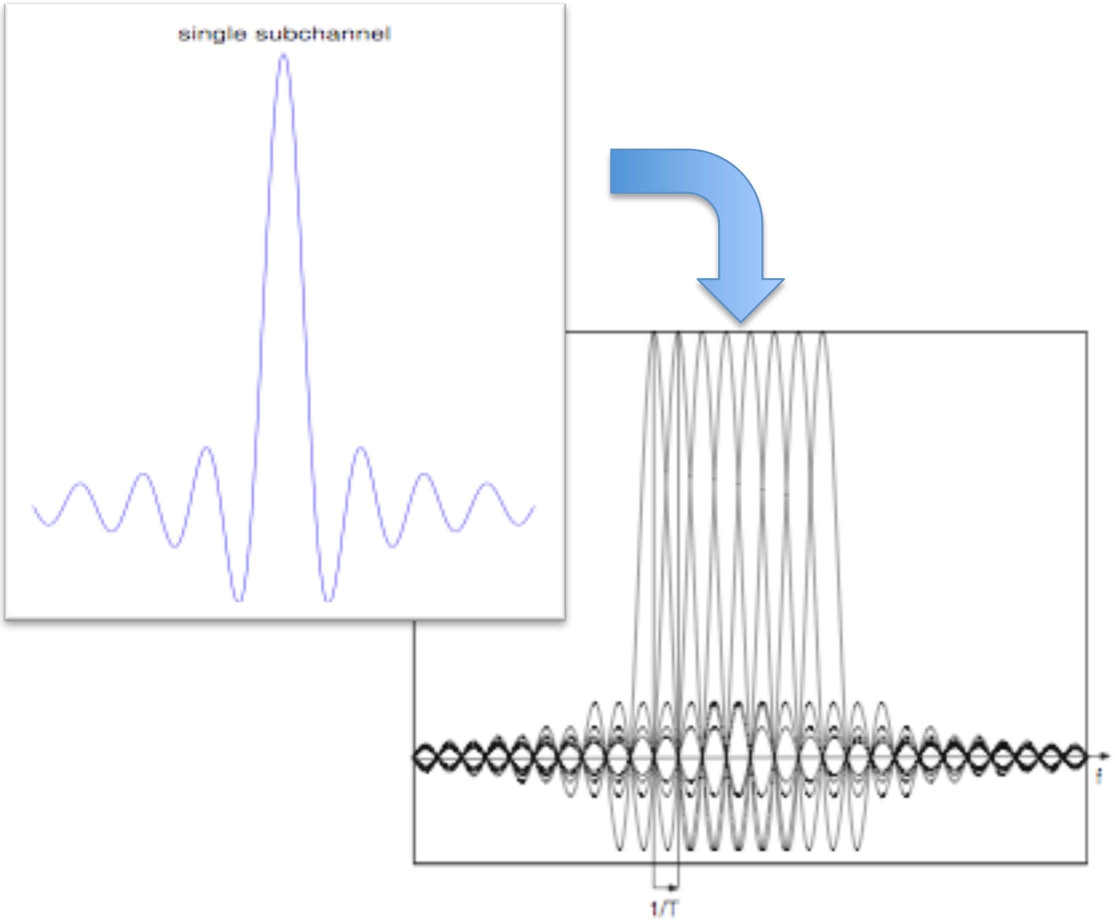
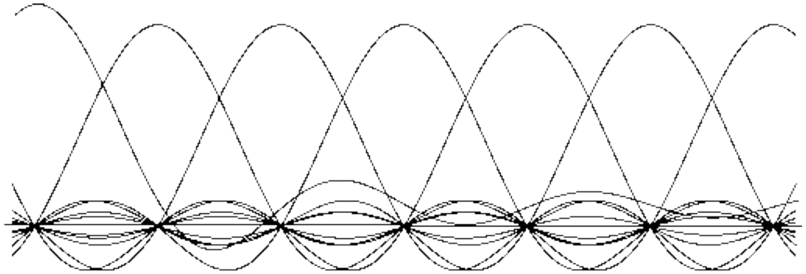
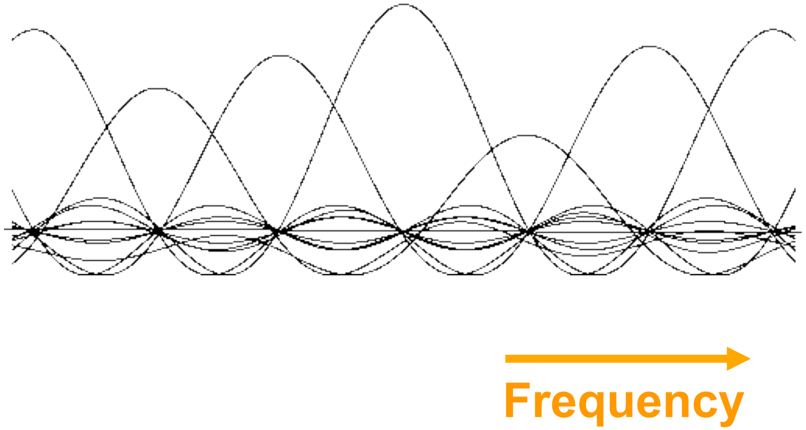


Fig. 4.5 OFDM spectrum for each QAM signal

OFDM Subcarrier Spectra



- before channel



- after channel

Pilots

- Transmitted instead of data – known by the receiver
  - Pilots are transmitted first in each burst
    - 802.11a/g uses 4 subchannels as pilots
    - Some timeslots can be used as pilots
    - Data can be normalized by pilot components
    - Pilots are designed for easy detection
    - Pilots are used for channel estimation
    - Frequency and phase offsets
    - Can be used for synchronization

Guard time: Cyclic Extension

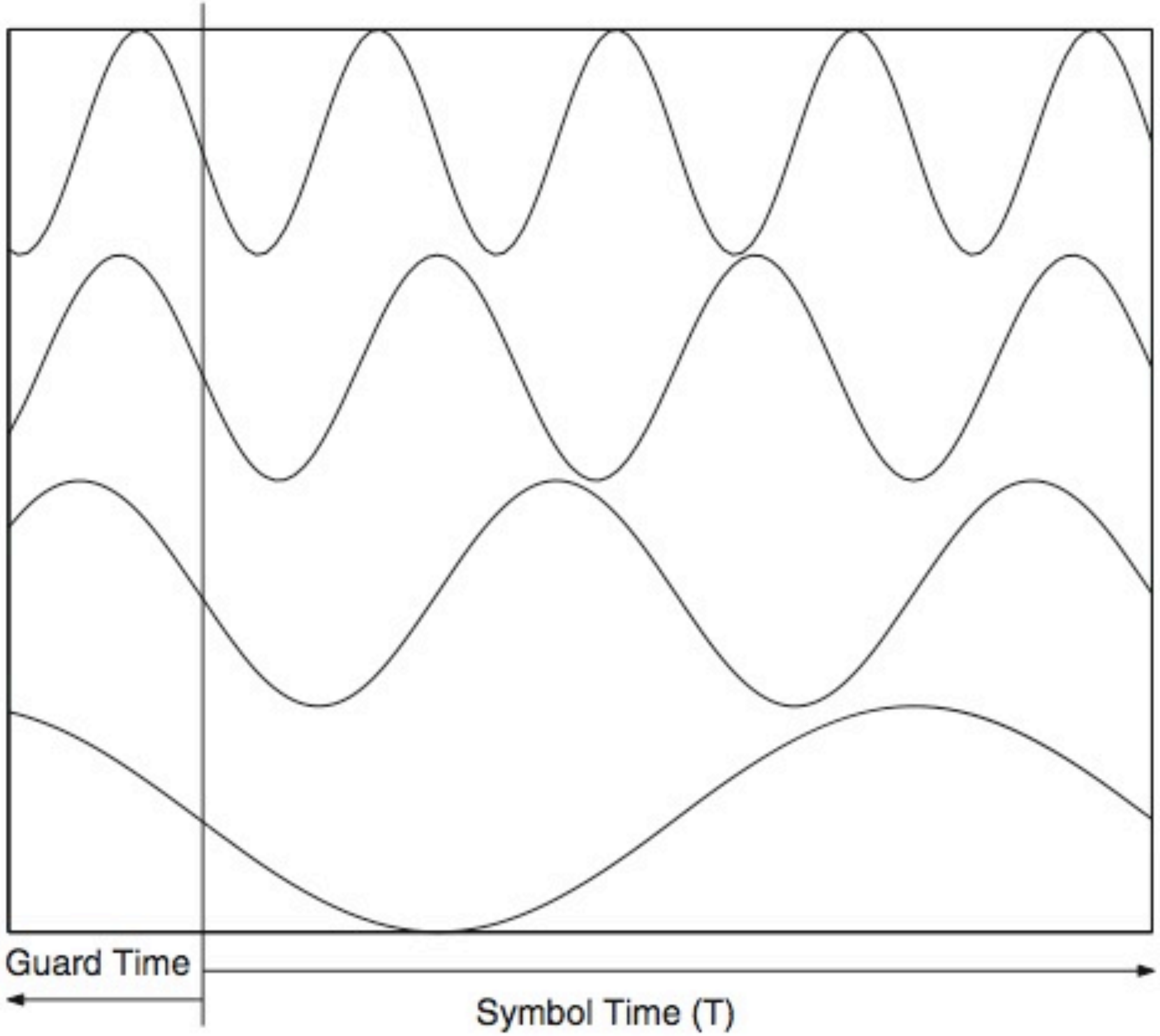
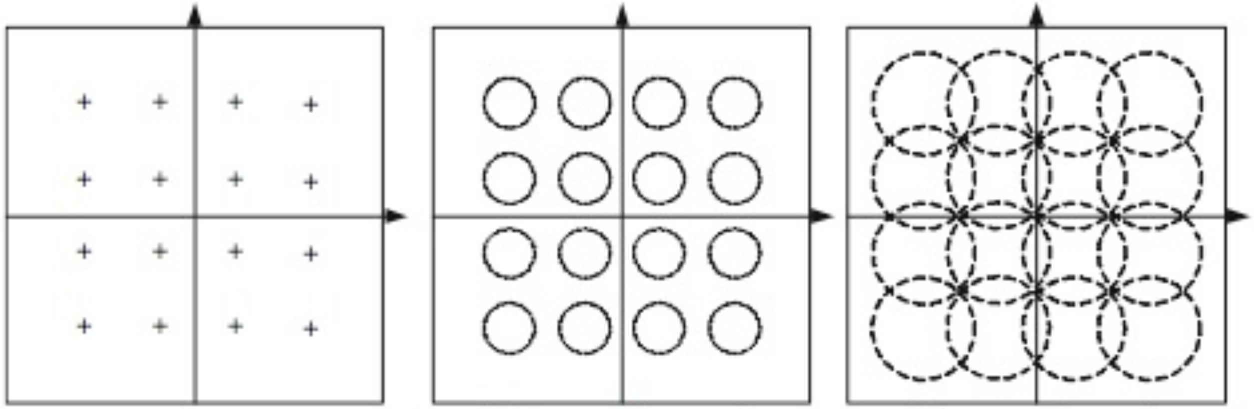


Fig. 4.6 OFDM with cyclic shift

Guard Time vs Delay spread



delay spread < guard time

delay spread exceeds guard time by 3% of the FFT interval

delay spread exceeds guard time by 10% of the FFT interval

Fig. 4.11 16QAM constellation

Guard Time increases the transmit energy and reduces the bit rate however is required to combat with ISI

A simplified OFDM system

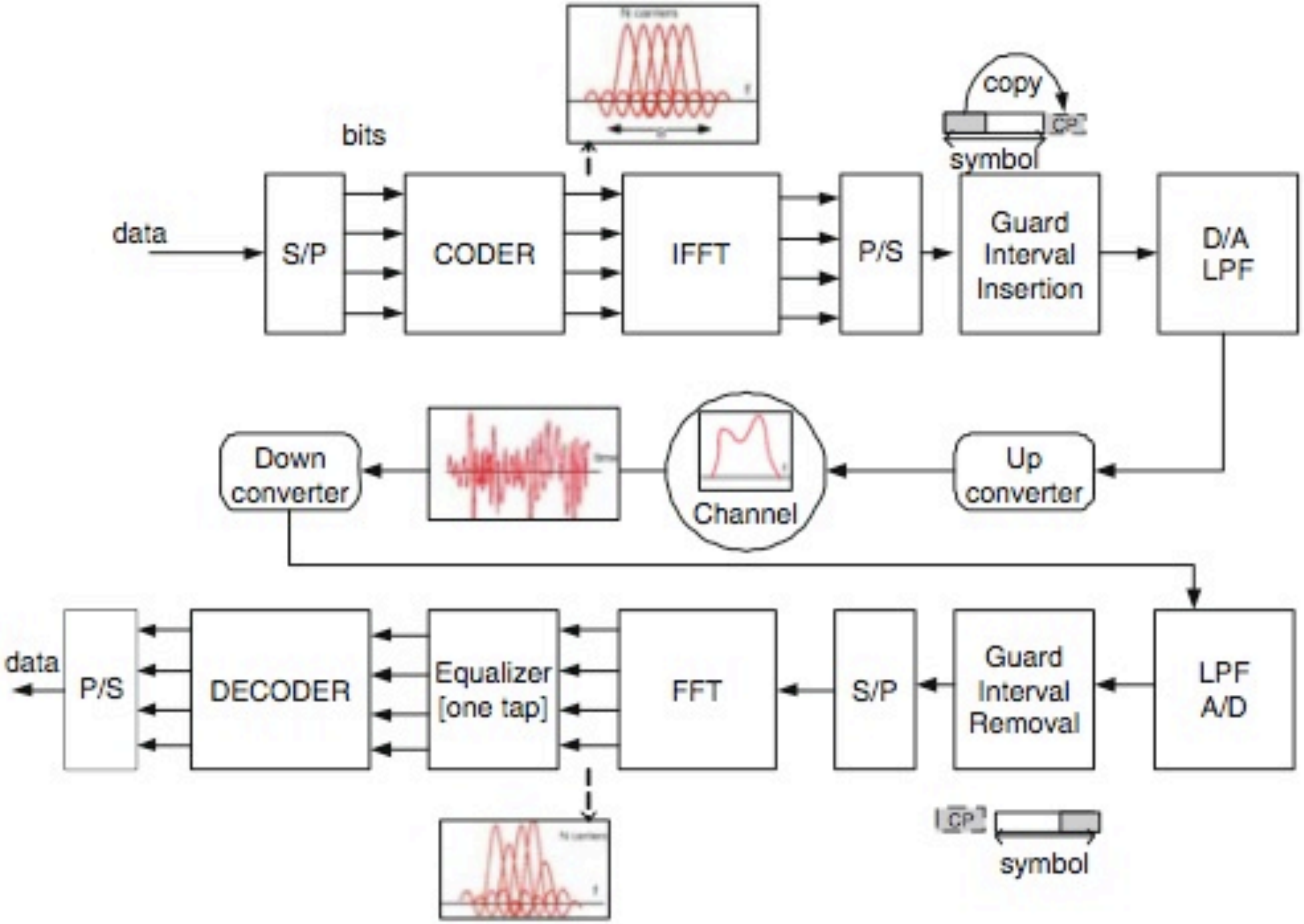


Fig. 4.8 Simplified OFDM system

Mechanics

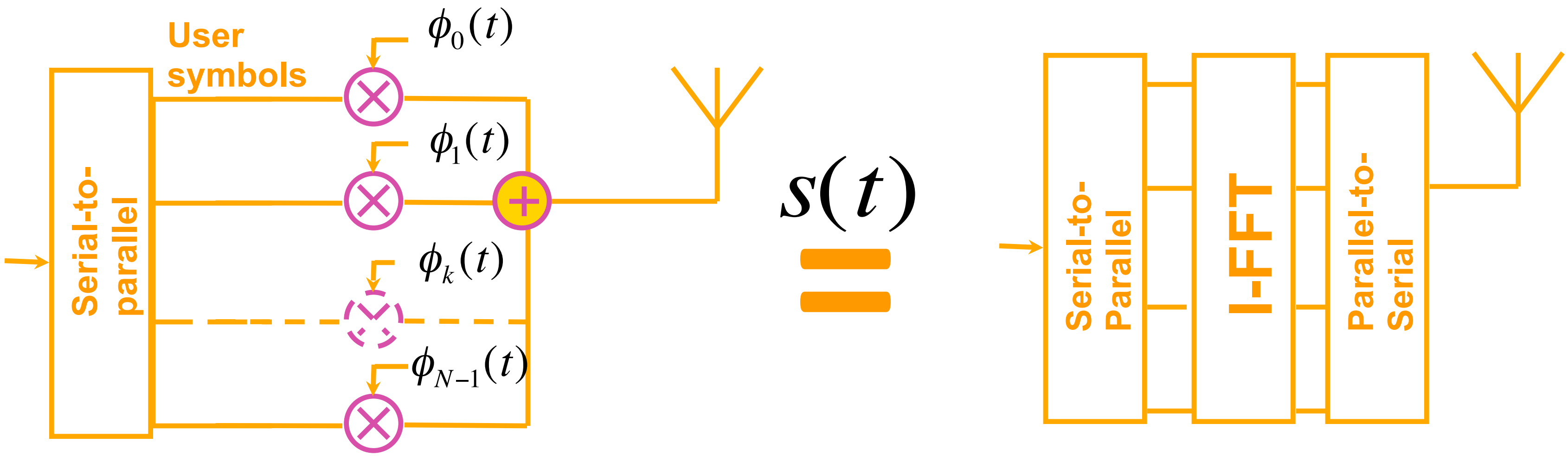
- The incoming data is converted from serial to parallel and grouped into bits each to form a complex number x after modulation in order to be transmitted over N low-rate data streams.
- Each low-rate data stream is associated with a subcarrier of the form

$$\phi_k(t) = e^{j2\pi f_k t}$$

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k \phi_k(t), \qquad 0 < t < T$$

I-FFT: OFDM Transmission

- Transmission of QAM symbols on parallel subcarriers
- Overlapping, yet orthogonal subcarriers



OFDM Symbol

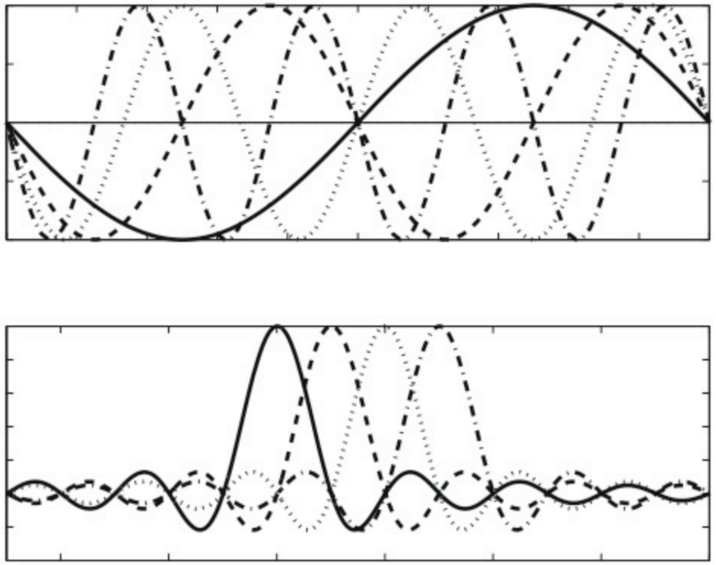


Fig. 4.9 An example of four subcarriers in time and frequency with same modulation

Same Modulation

Different Modulation

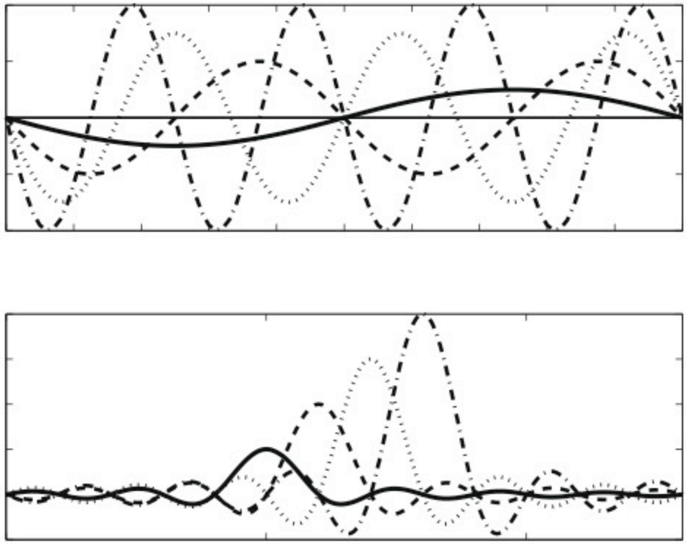


Fig. 4.10 An example of four subcarriers in time and frequency with different modulation: Modulation level increases with the increasing number of subcarriers

Orthogonality of OFDM subcarriers in frequency domain is with Dirac pulses convolved with $\mathrm{sinc}(\pi f T)$. Since in time domain a subcarrier ϕ_k is multiplied with a rectangle(T), which is in frequency domain a convolution between $\delta(f - f_k)$ and $\mathrm{sinc}(\pi f T)$. This is basically $1/T$ shifted version of $\mathrm{sinc}(f)$ for each f_k and $\mathrm{sinc}(\pi f T)$ has zeros for all frequencies that are integer multiple of $1/T$.

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Received Signal

Received Signal

Number of Subcarriers

$$y(t) = s(t) * h(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} H_k x_k \phi_k(t), \qquad 0 < t < T$$

Delay Spread

$$H_k = \int_0^{\tau_h} h(t) e^{j2\pi f_k t} dt$$

Channel Impulse Response

$$y_k = H_k x_k, \qquad k = 0,\ldots,N-1$$

Received signal after sampling

$$\hat{x}_k = \frac{y_k}{\hat{H}_k}$$

Channel estimation

Estimated Input

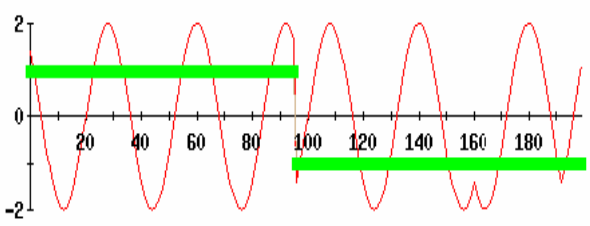
Example of OFDM

- Lets we have following information bits
  - 1, 1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, …
- Just converts the serials bits to parallel bits

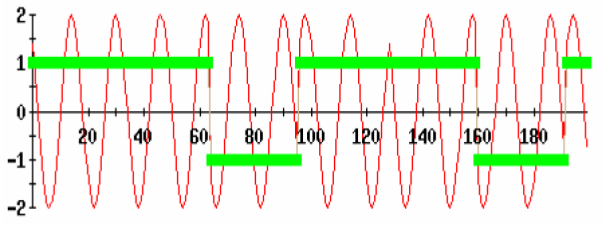
C1	C2	C3	C4
1	1	-1	-1
1	1	1	-1
1	-1	-1	-1
-1	1	-1	-1
-1	1	1	-1
-1	-1	1	1

Example of OFDM cont..

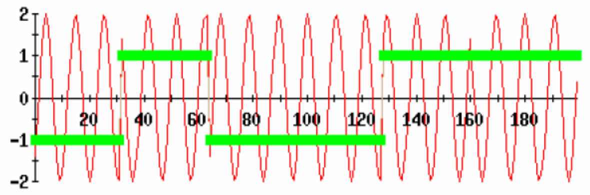
Modulate each column with corresponding sub-carrier using BPSK



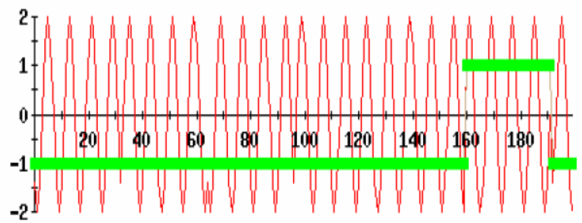
Modulated signal for C1



Modulated signal for C2



Modulated signal for C3



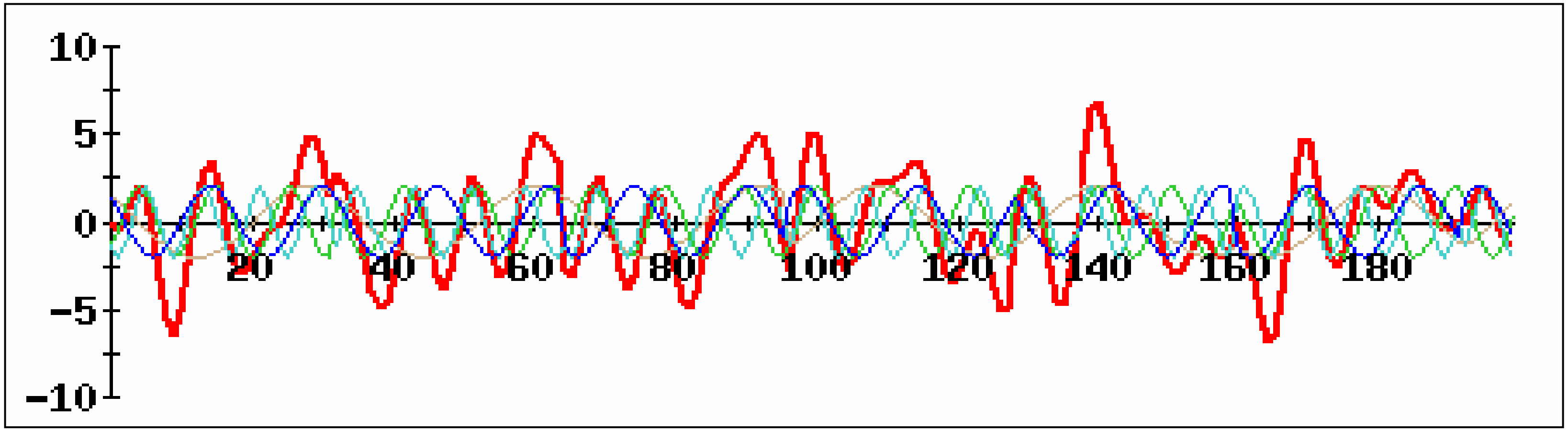
Modulated signal for C4

Example of OFDM cont..

- Final OFDM Signal = Sum of all signal

V(t) ———

$$V(t) = \sum_{n=0}^{N-1} I_n(t)\sin(2\pi nt)$$



Generated OFDM signal, V(t)

OFDM Architecture

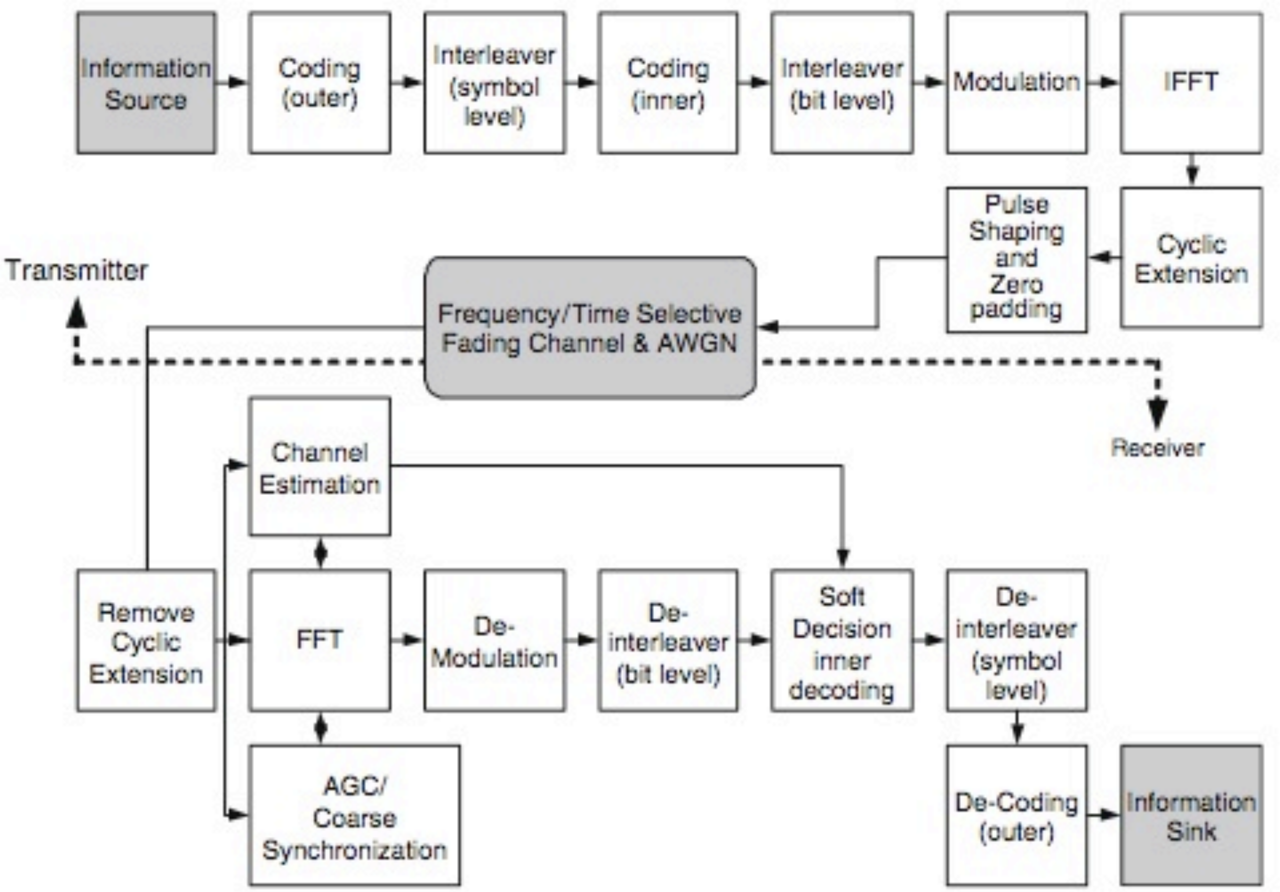
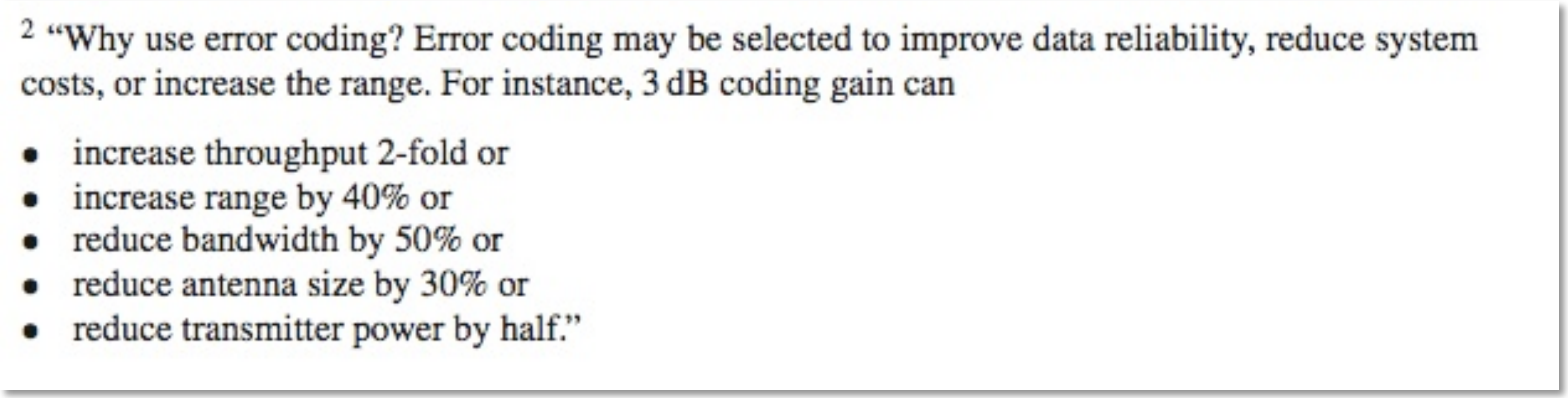


Fig. 4.12 A typical wireless OFDM architecture

Coding

- In a multipath environment, all subcarriers will experience different fading environment and all will arrive with different amplitudes.
  - Block coding
  - Convolutional coding
  - Concatenated coding
  - Turbo coding
  - Trellis coding

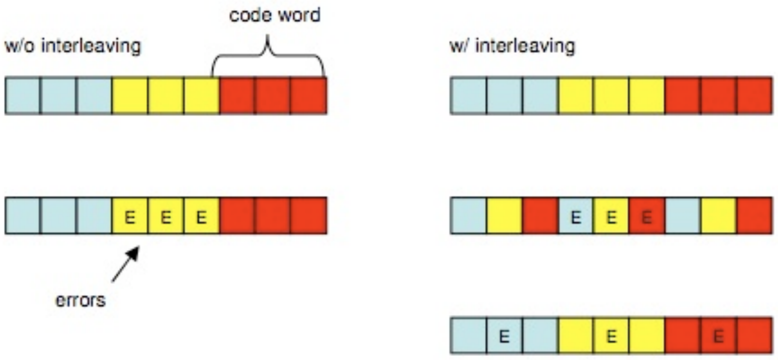


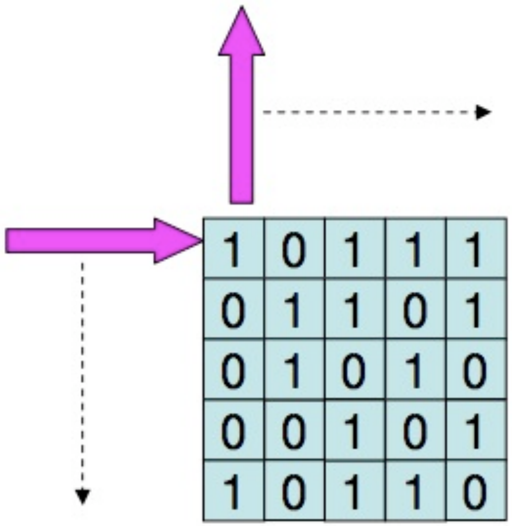
[2] "Why use error coding? Error coding may be selected to improve data reliability, reduce system costs, or increase the range. For instance, 3 dB coding gain can

- increase throughput 2-fold or
- increase range by 40% or
- reduce bandwidth by 50% or
- reduce antenna size by 30% or
- reduce transmitter power by half."

Interleaving

- Interleaving
  - Scatters error bursts
  - Can be done in time or in frequency domain
  - Block Interleaving
    - Write row-by-row
    - Read column-by-column
    - Additional matrix permutation is possible





1	0	1	1	1
0	1	1	0	1
0	1	0	1	0
0	0	1	0	1
1	0	1	1	0

Block coding

- A way of mapping *k* symbols to *n* symbols with *n>k*
- We call *k* symbols a messageword,
- and the block of *n* symbols a codeword.
- Mapping *k* message symbols to *n* code symbols is called encoding,
- and the reverse process is called decoding.

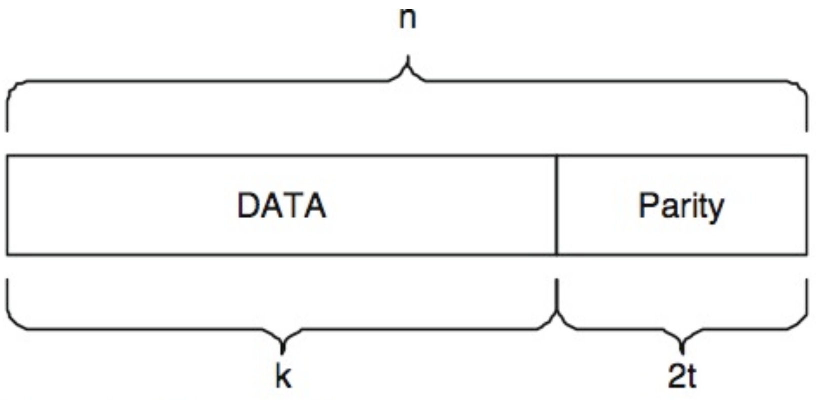


Fig. 4.14 Construction of a systematic block code

Systematic if messageword is directly recovered by removing the parity bit.

Linear if the sum of two codewords is always a valid codeword, and a scalar multiple of any codeword is also a valid codeword.

Cyclic Redundancy Check

- Error detection
  - CRC-n can detect errors less than n with a probability of $1 - 2^{-n}$

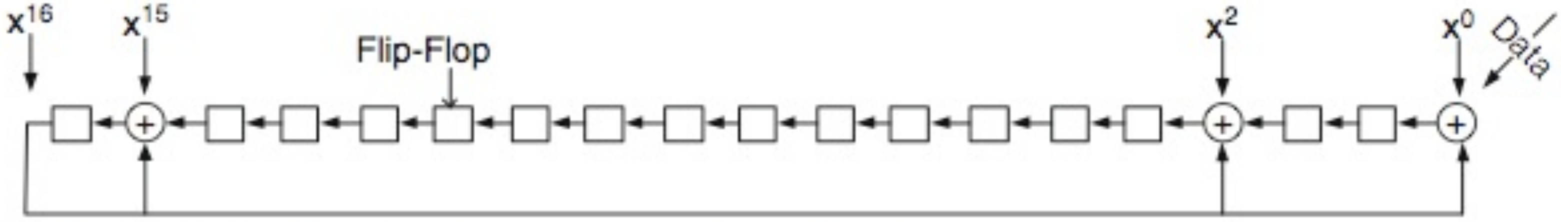
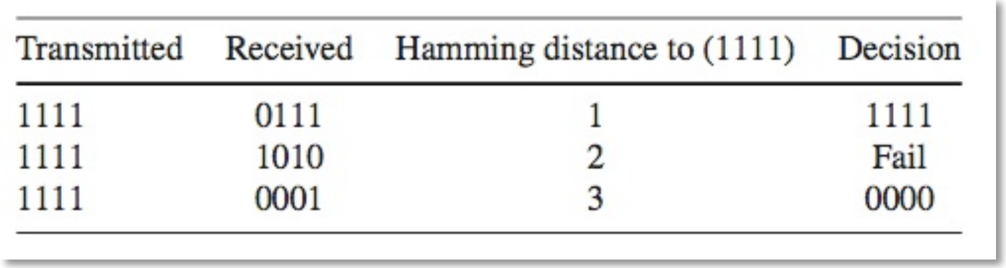


Fig. 4.16 CRC-16 implementation: $P(x) = x^{16} + x^{15} + x^{2} + 1$

Error Correction

- Hamming weight
  - Number of nonzero symbols in codeword
    - Codeword is 122200001 then Hamming weight is 5.
- Hamming distance
  - Comparison metric between two codewords by the number of places where the codewords differ
    - Codewords 100111 and 110000, the Hamming distance is 4.
- Minimum distance of a code
  - Lowest Hamming distance in linear codewords

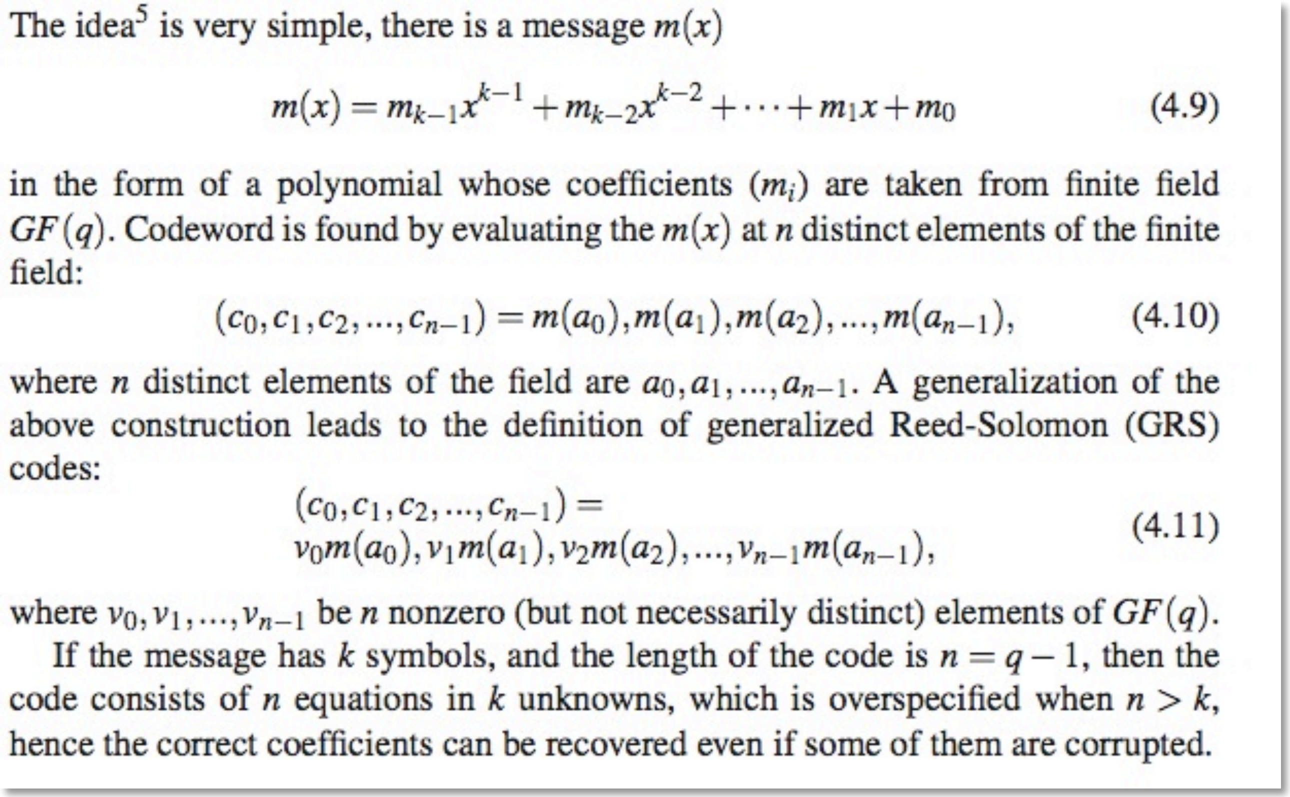


Transmitted	Received	Hamming distance to (1111)	Decision
1111	0111	1	1111
1111	1010	2	Fail
1111	0001	3	0000

For a minimum distance d, we can correct up to distance floor ((d-1)/2)

Reed-Solomon Coding

- Popular coding scheme of today
- Cyclic code and linear code.
- Cyclic codewords are still a codeword when shifted.



The idea[5] is very simple, there is a message $m(x)$

$$m(x) = m_{k-1}x^{k-1} + m_{k-2}x^{k-2} + \cdots + m_1 x + m_0 \tag{4.9}$$

in the form of a polynomial whose coefficients (m_i) are taken from finite field $GF(q)$. Codeword is found by evaluating the $m(x)$ at n distinct elements of the finite field:

$$(c_0, c_1, c_2, \ldots, c_{n-1}) = m(a_0), m(a_1), m(a_2), \ldots, m(a_{n-1}), \tag{4.10}$$

where n distinct elements of the field are $a_0, a_1, \ldots, a_{n-1}$. A generalization of the above construction leads to the definition of generalized Reed-Solomon (GRS) codes:

$$\begin{aligned}&(c_0, c_1, c_2, \ldots, c_{n-1}) = \\ &v_0 m(a_0), v_1 m(a_1), v_2 m(a_2), \ldots, v_{n-1} m(a_{n-1}),\end{aligned} \tag{4.11}$$

where $v_0, v_1, \ldots, v_{n-1}$ be n nonzero (but not necessarily distinct) elements of $GF(q)$.

If the message has k symbols, and the length of the code is $n = q-1$, then the code consists of n equations in k unknowns, which is overspecified when $n > k$, hence the correct coefficients can be recovered even if some of them are corrupted.

Reed-Solomon Coding

- The best minimum distance obtainable codes.
- They are linear, cyclic and their generator polynomial has well-defined roots (2t).
- Easy to encode and decode
- A popular Reed-Solomon code is RS(255,223) with 8-bit symbols where n=255 and k=223 and s=8 and 2t=32. The decoder can correct any 16 symbol errors in the codeword.

This is a very popular value because of the prevalence of byte-oriented computer systems.

Reed-Solomon code

- The Reed–Solomon code, like the convolutional code, is a transparent code.
- This means that if the channel symbols have been inverted somewhere along the line, the decoders will still operate.
- The result will be the inversion of the original data. However, the Reed–Solomon code loses its transparency when the code is shortened.
- The "missing" bits in a shortened code need to be filled by either zeros or ones, depending on whether the data is complemented or not.
- (To put it another way, if the symbols are inverted, then the zero-fill needs to be inverted to a one-fill.)
- For this reason it is mandatory that the sense of the data (i.e., true or complemented) be resolved before Reed–Solomon decoding.

Convolutional Coding

- Operates on serial streams rather than blocks.

- Described by two parameters: the code rate *(k/n)* and constraint length (*K*) – length of the convolutional encoder which denotes the number of stages and the cycles an input bit retains in the convolutional encoder.

- Viterbi decoding or sequential decoding are used.
- Viterbi decoding has fixed decoding time – it is preferable in hw implementation, allows soft decision decoding.

Convolutional coding

- Encoder
  - K=3 (7,5), code rate is ½ and m=2.
  - (7,5) is code generator polynomials

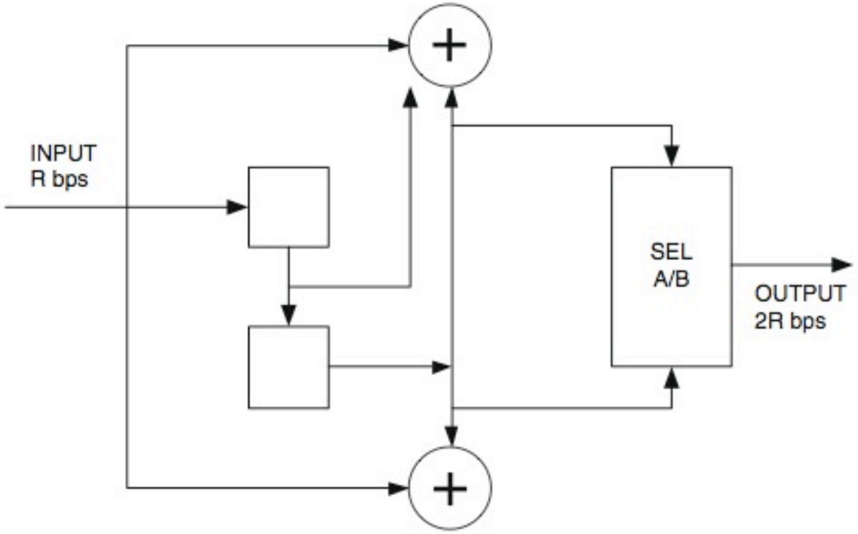


Fig. 4.17 Generation of a convolutional code: data bits are provided at a rate of *R* bits per second. Channel symbols are output at a rate of 2*R* symbols per second. The input bit is stable during the encoder cycle. When the input clock edge occurs, the output of the left-hand flip–flop is clocked into the right-hand flip–flop, the previous input bit is clocked into the left-hand flip–flop, and a new input bit becomes available. Then the outputs of the upper and lower modulo-two adders become stable. The output selector (SEL A/B block) cycles through two states-in the first state, it selects and outputs the output of the upper modulo-two adder; in the second state, it selects and outputs the output of the lower modulo-two adder

Convolutional coding

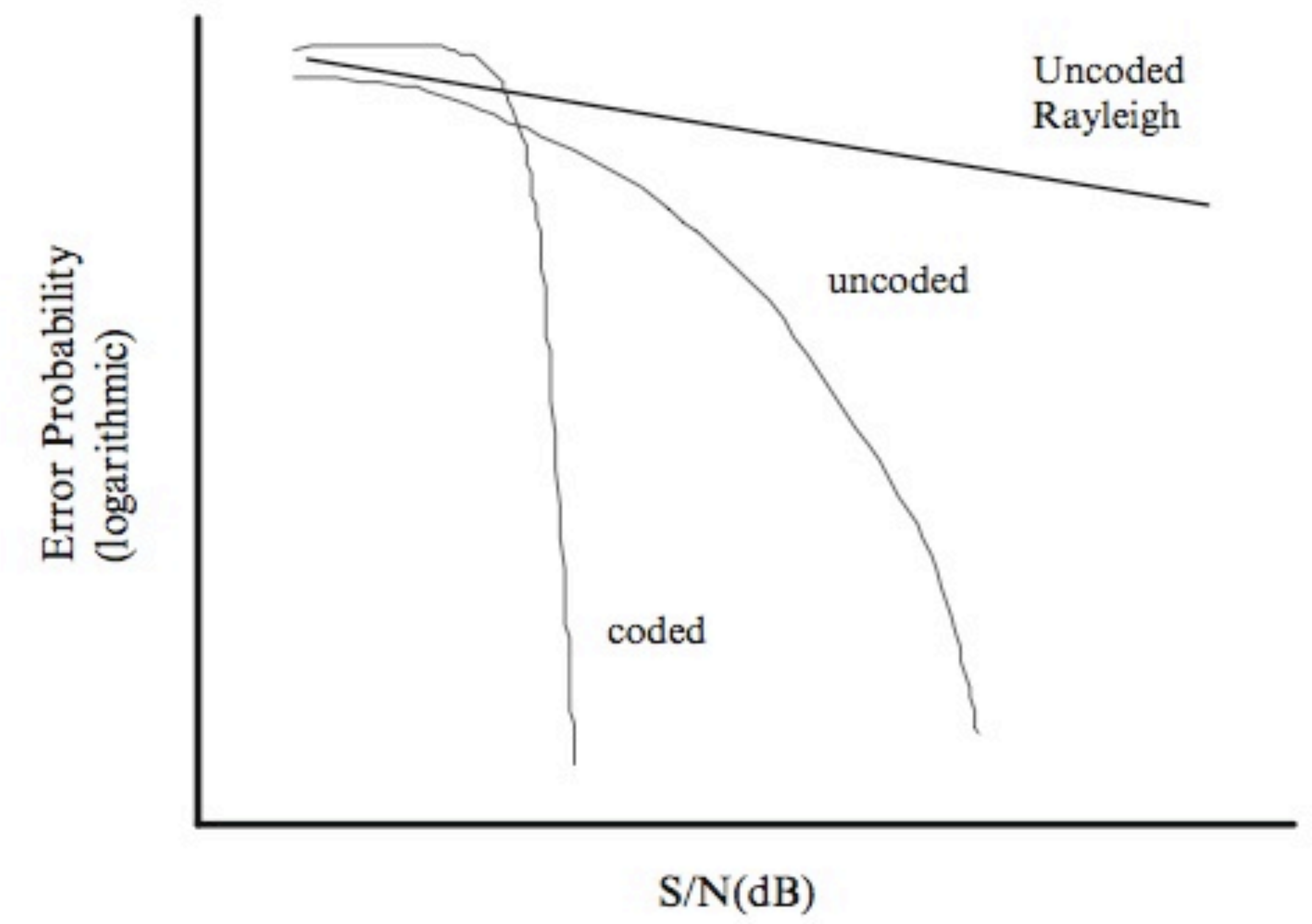


Fig. 4.19 Performance of a convolutional code over a Rayleigh fading channel

Viterbi Decoder

- Viterbi decoder decides about the original bits according to accumulated error metric. At time instant *t*, the smallest accumulated error metric is selected according to the history of what states preceded the states. This method is called *traceback* method.

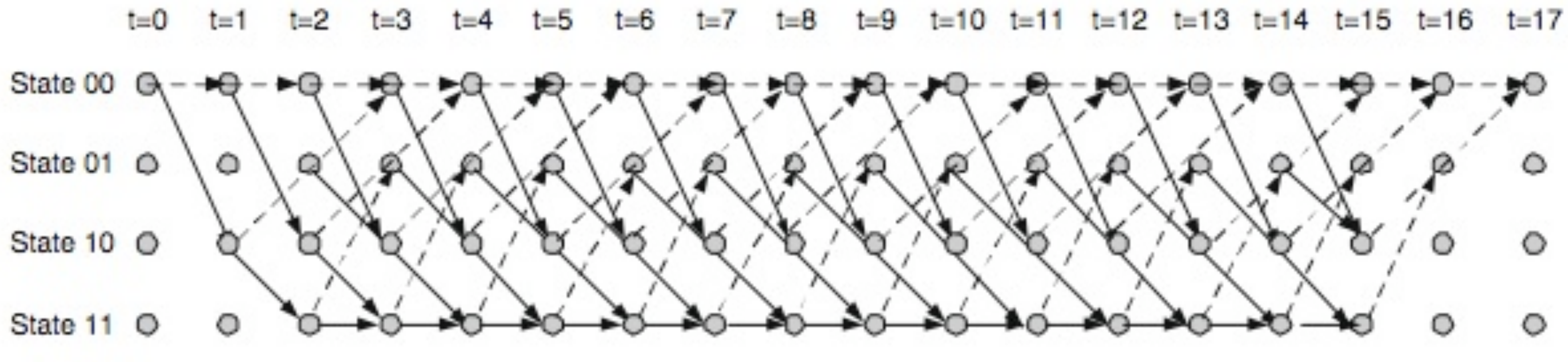
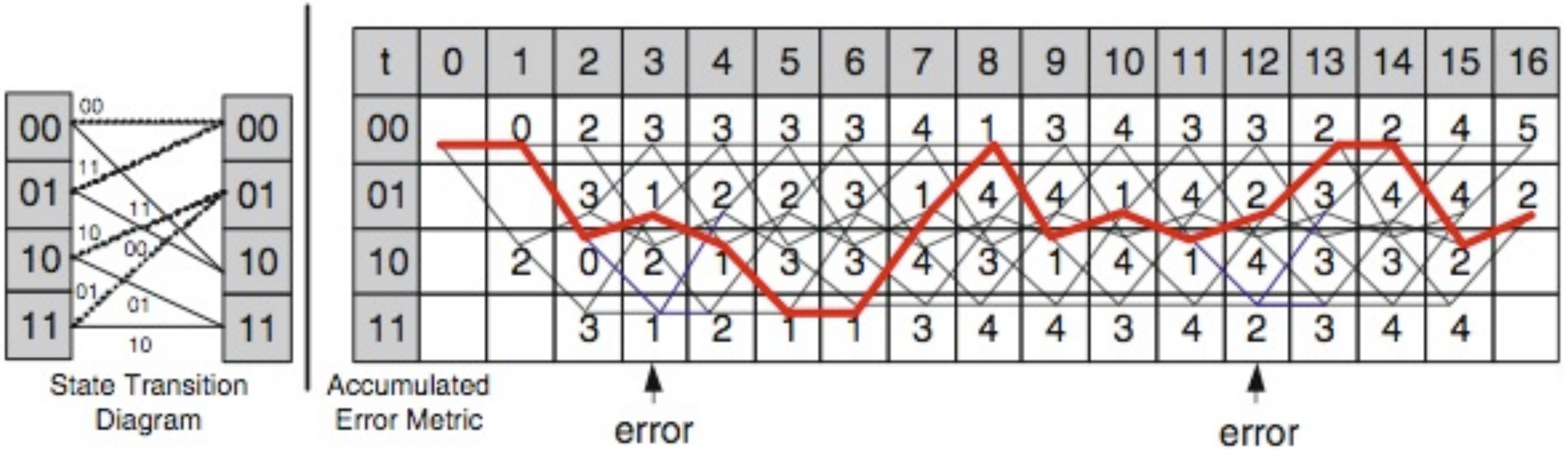


Fig. 4.20 Trellis diagram

Viterbi decoding

- For each branch, there is accumulated error metric and minimum is selected. If accumulated error metric is equal, then the decoder decides by looking forward. For instance, at time *t =3* and *t=12*, there are errors, and the decoder can end up with the same accumulated error.



State Transition Diagram

Accumulated Error Metric

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
00		0	2	3	3	3	3	4	1	3	4	3	3	2	2	4	5
01			3	1	2	2	3	1	4	4	1	4	2	3	4	4	2
10		2	0	2	1	3	3	4	3	1	4	1	4	3	3	2	
11			3	1	2	1	1	3	4	4	3	4	2	3	4	4	

Fig. 4.21 Viterbi decoding process

Concatenated coding

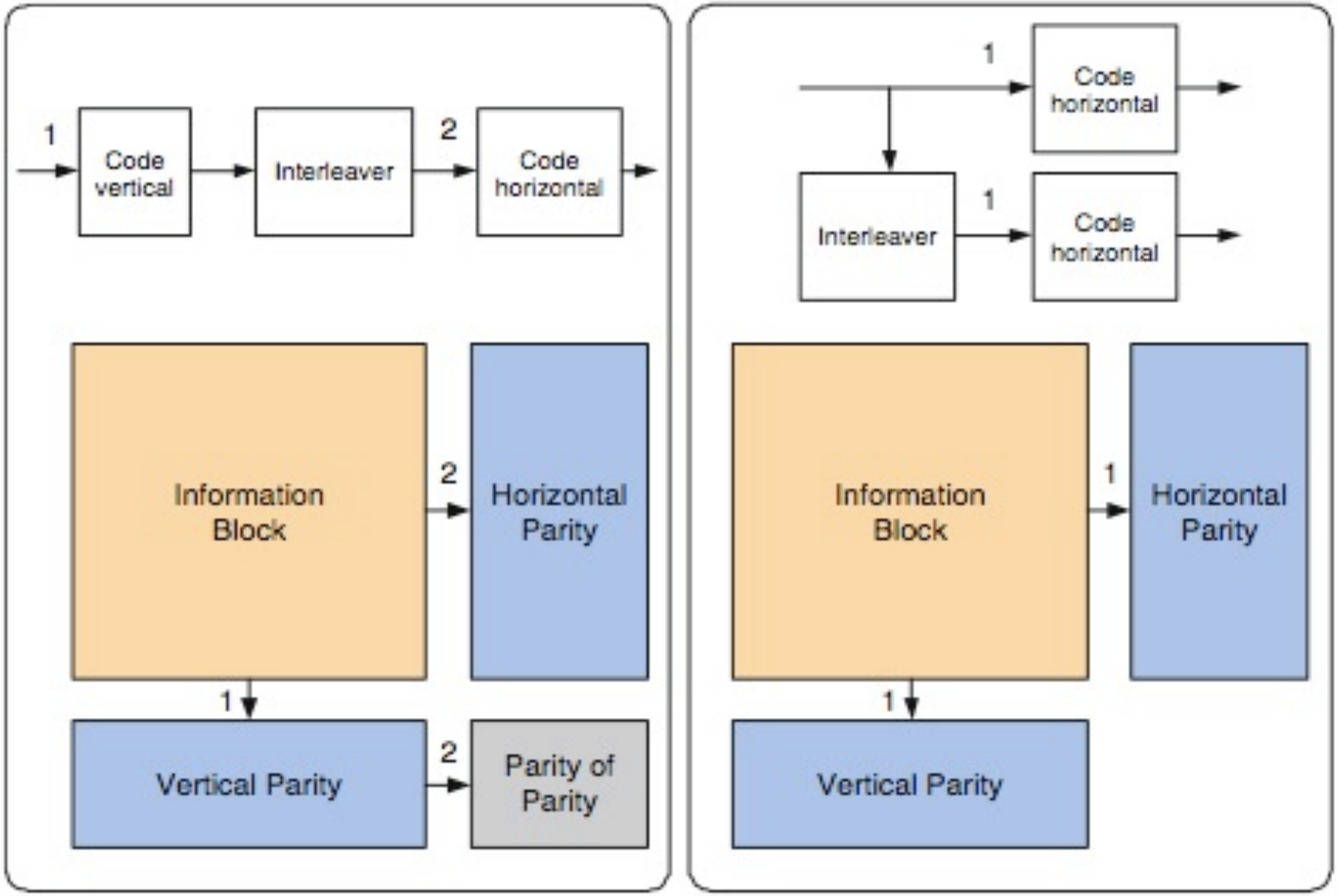


Fig. 4.22 Concatenated coding

Concatenated coding

- The inner convolutional code performs superior error correction with soft decision decoding, and if convolutional code makes an error, it causes a large burst, since Viterbi algorithm may pick a wrong sequence. In this case, we know that block coding, especially an interleaved Reed-Solomon coding, is superior correcting the bursty errors.

Concatenated coding provides means to constructing long codes, and it also confirms Shannon's channel coding theorem by stating that if the code is long enough, any error can be corrected.

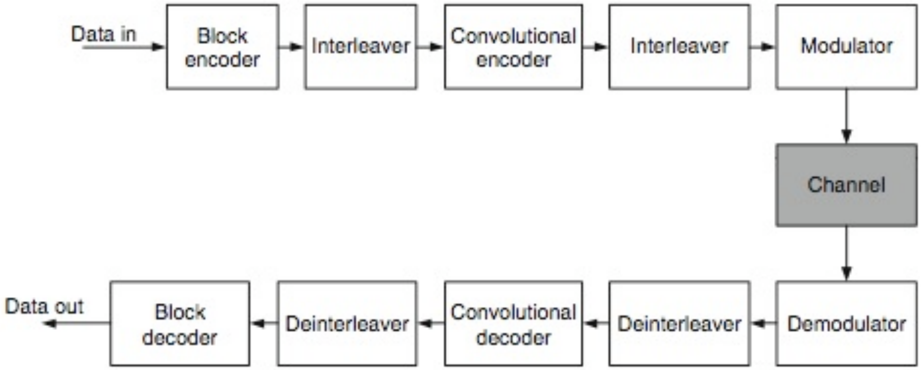


Fig. 4.23 Concatenated coding with interleaving

Trellis Coding

- Combination of coding and modulation.
- Trellis coding adds redundant constellation points rather than redundant bits or symbols. Consequently, bit rate increases but the symbol rate stays the same and it conserves bandwidth. Increasing the constellation size reduces Euclidean distance between the constellation points, but sequence coding offers a coding gain that over- comes the power disadvantage of going to higher constellation.

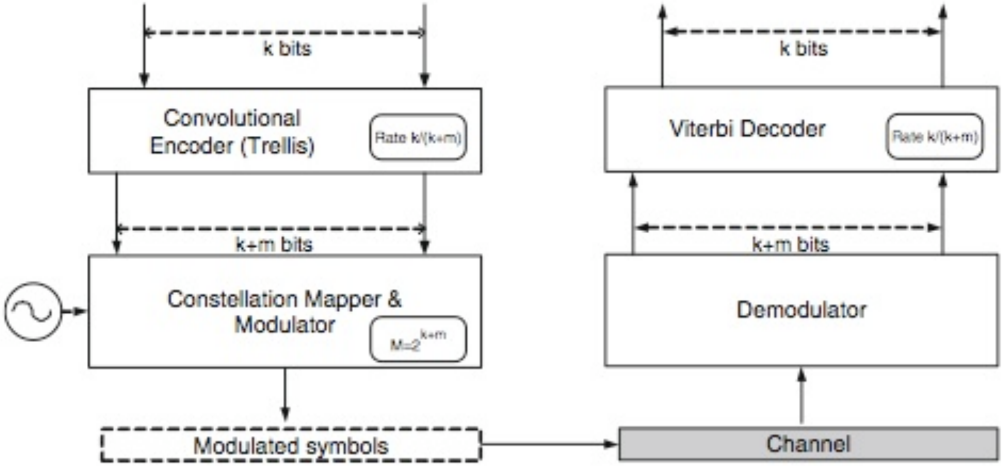


Fig. 4.24 Trellis coded modulation

Trellis coding

Adds one extra bit and expands the constellation without increasing the signal energy. The signal energy is kept the same, since the distance between the symbols decreases. Although it sounds like a disadvantage, advantage comes from the restriction on what transitions are allowed in the constellation.

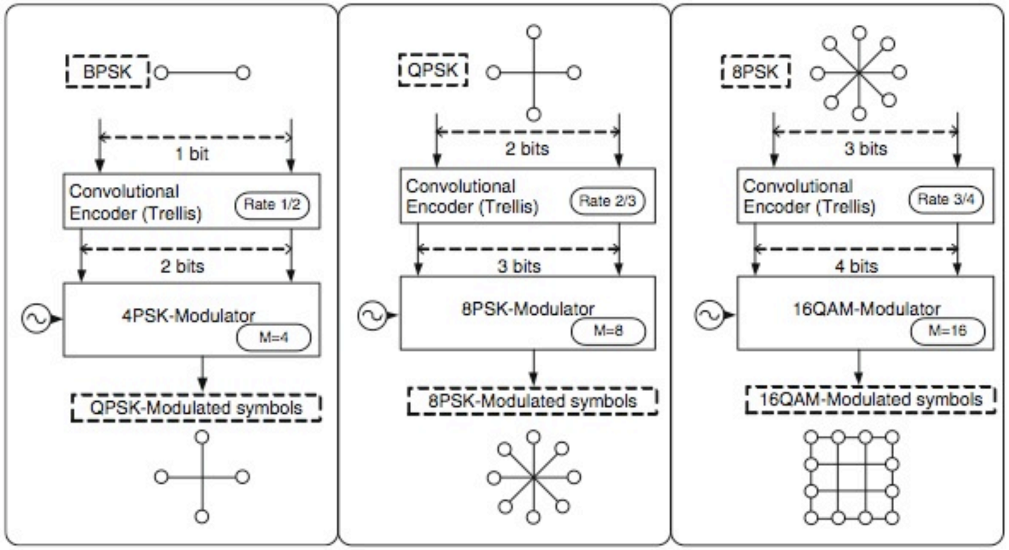


Fig. 4.25 Trellis-coded modulation: BPSK: code rate 1/2, output QPSK; QPSK code rate 2/3, output 8PSK; 8PSK, code rate = 3/4, output 16QAM

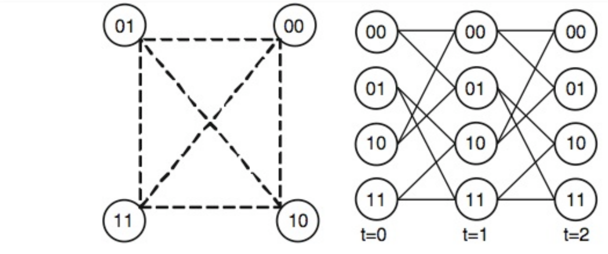


Fig. 4.26 QPSK with and without Trellis coding

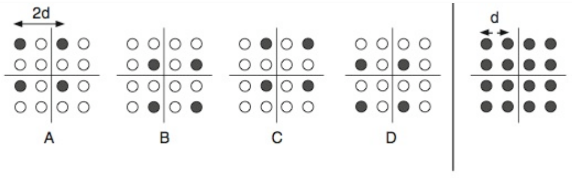


Fig. 4.27 Subsets in the constellation for 16-point

Turbo coding

RSC is (1,G2/G1) but Convolutional codes are (G1,G2)

- A typical turbo coding11 includes parallel concatenated convolutional codes where the information bits are coded by two or more recursive systematic convolutional (RSC) codes, which are typically interleaved and optionally punctured

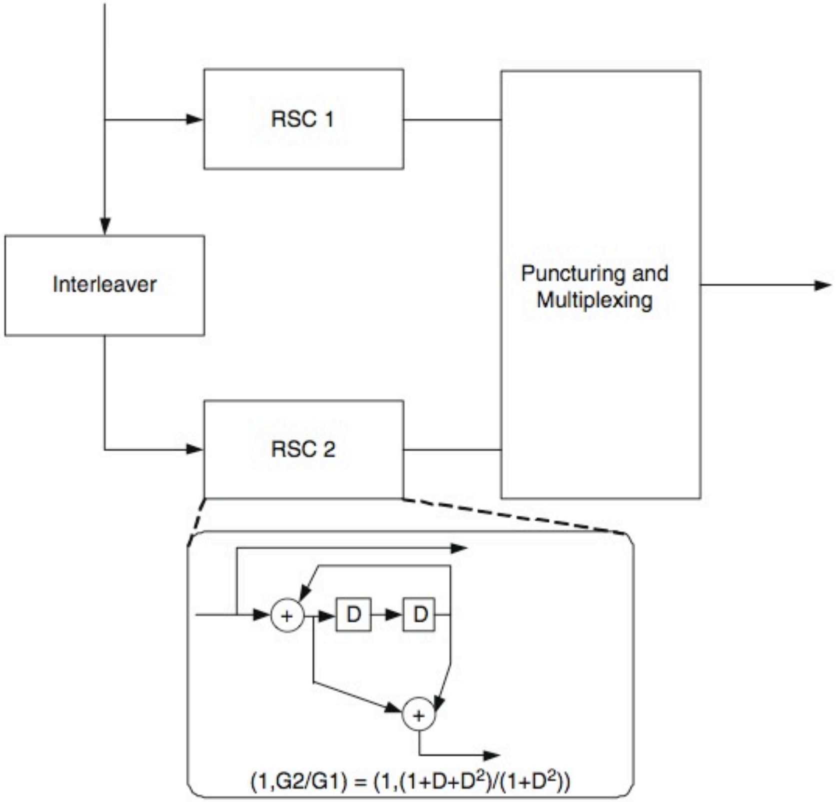


Fig. 4.28 A typical turbo encoder: two identical 1/2 RSC encoder separated by an N-bit interleaver and optional puncturing

Turbo coding

- Turbo codes increase data rate without increasing the power of a transmission, or they can be used to decrease the amount of power used to transmit at a certain data rate.
- Turbo coding shows high error correction performance because of its structure based on interleaving in conjunction with concatenated coding and iterative decoding using (almost) uncorrelated extrinsic information.
- Turbo coding such as block turbo coding and convolutional turbo coding are included in IEEE 802.16 as sup- ported coding schemes.

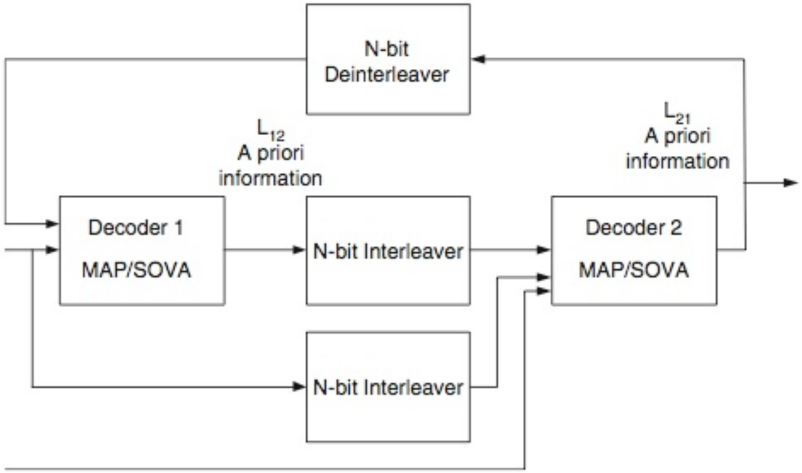


Fig. 4.29 A turbo decoder structure that uses two decoders operating cooperatively

LPDC Coding (Low-density parity check)

- A (*n*,*k*) LDPC encoder would have an **H** matrix, which is *mxn* in size where *m=n-k*.

$$\begin{pmatrix} 01011001 \\ 11100100 \\ 00100111 \\ 10011010 \end{pmatrix}$$

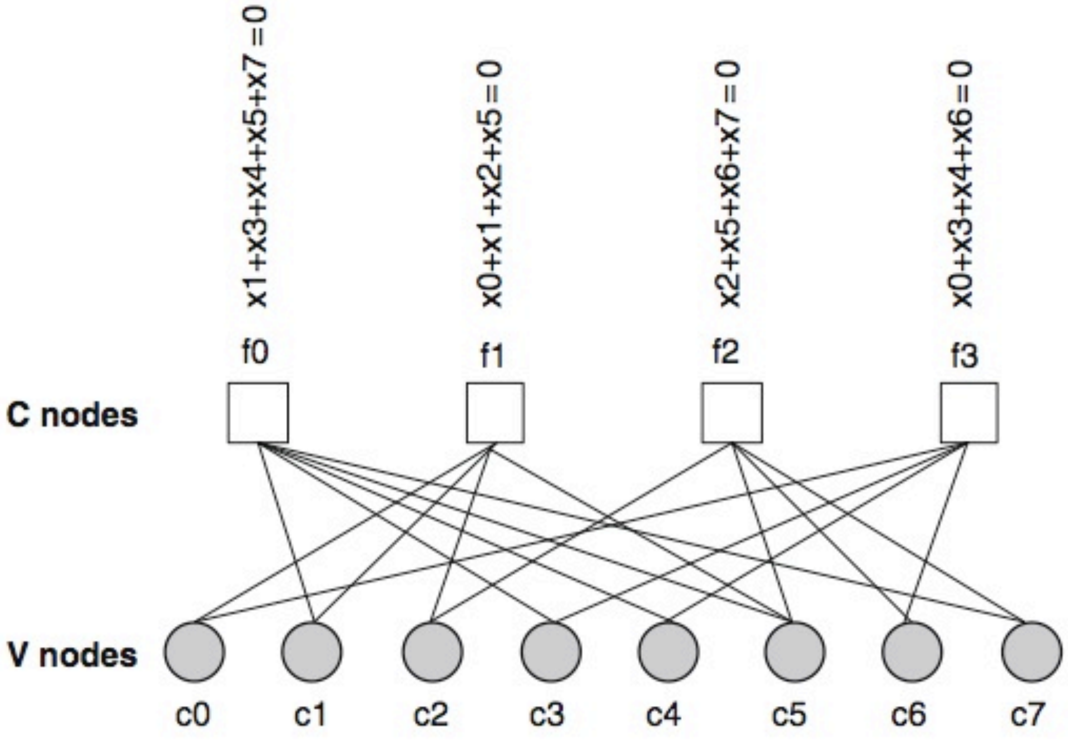


Fig. 4.30 Graphical representation of (8,4) LMDS

LDPC decoding

- Hard decision decoding

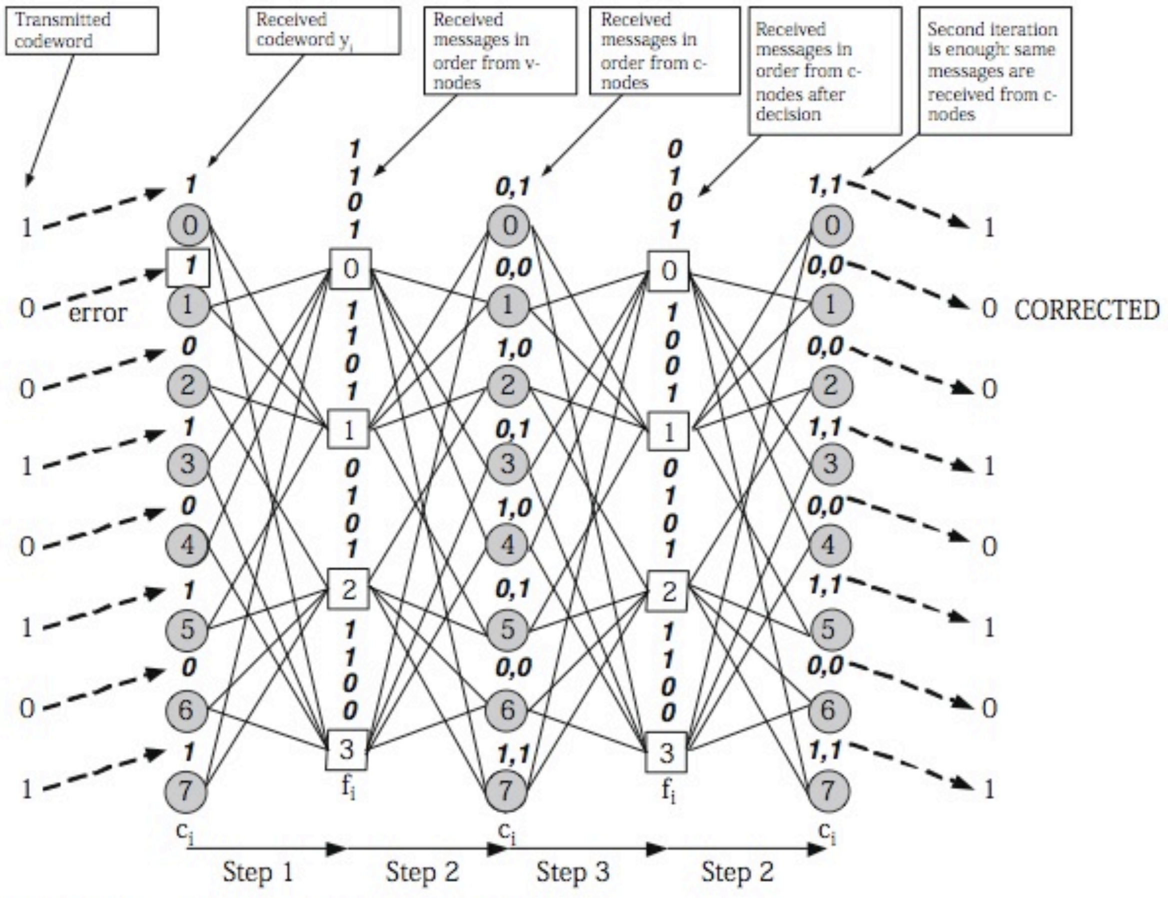


Fig. 4.31 Hard decision decoding for LDPC

Synchronization

- OFDM needs to employ time and frequency synchronization.
  - Time synchronization is to decide for the symbol boundaries. Commonly, a sequence of known symbols preamble are used to detect the symbol boundaries.
  - It has less sensitivity to timing offset as compared with singlecarrier systems since timing offset does not violate the orthogonality of subcarriers in OFDM system, but causes ISI in single-carrier systems.
  - Unlike time synchronization, frequency synchronization, which is to estimate the frequency offset in the oscillators in order to align the oscillators in the transmitter and receiver, is essential otherwise ICI occurs, since subcarriers could be shifted from its original position and the receiver may experience non-orthogonal signals.

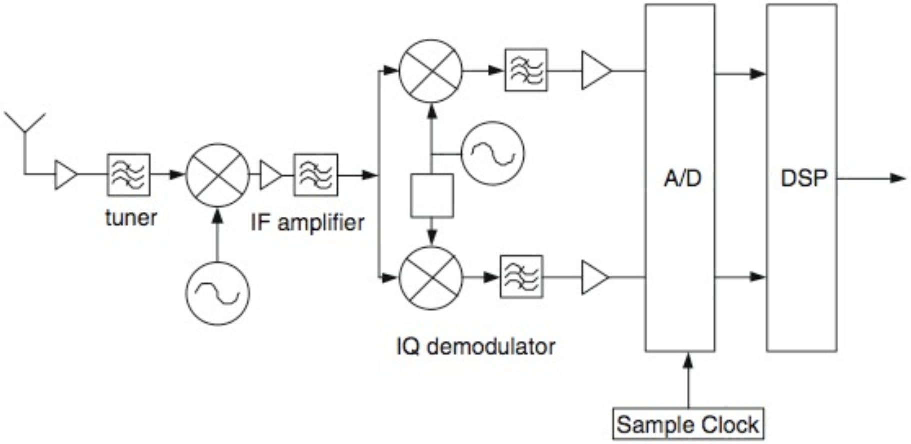


Fig. 4.32 Front end of an OFDM receiver

Synchronization

- Timing Offset
  - OFDM is insensitive to timing offset as long as offset is within the guard time.
    - No ISI and ICI is guaranteed.

- Frequency Offset
  - OFDM is sensitive to the frequency offset since it causes ICI.

Pilot-assisted Time/Frequency Synchronization

- Pilots are transmitted – known at the receiver.
  - Symbol timing and carrier frequency offset can be estimated at the receiver.
  - Frame Synchronization by preamble- known pilot symbols at the beginning of the frame.

There is also blind time-frequency synchronization: It is pilotless and based on maximum likelihood estimation. The parameters that need to be estimated requires longer observation.

Detection and Channel Estimation

- To estimate the transmitted bits at the receiver, channel knowledge is required in addition to the estimates of random phase shift and amplitude change, caused by carrier frequency offset and timing offset.

Coherent Detection

- Channel variation in time and frequency are used to determine the minimum pilot spacing in time and frequency.

Pilot spacing in time

$$N_p^t \approx \frac{1}{B_D T} \qquad N_p^f \approx \frac{1}{\Delta f T_S},$$

Pilot spacing in frequency

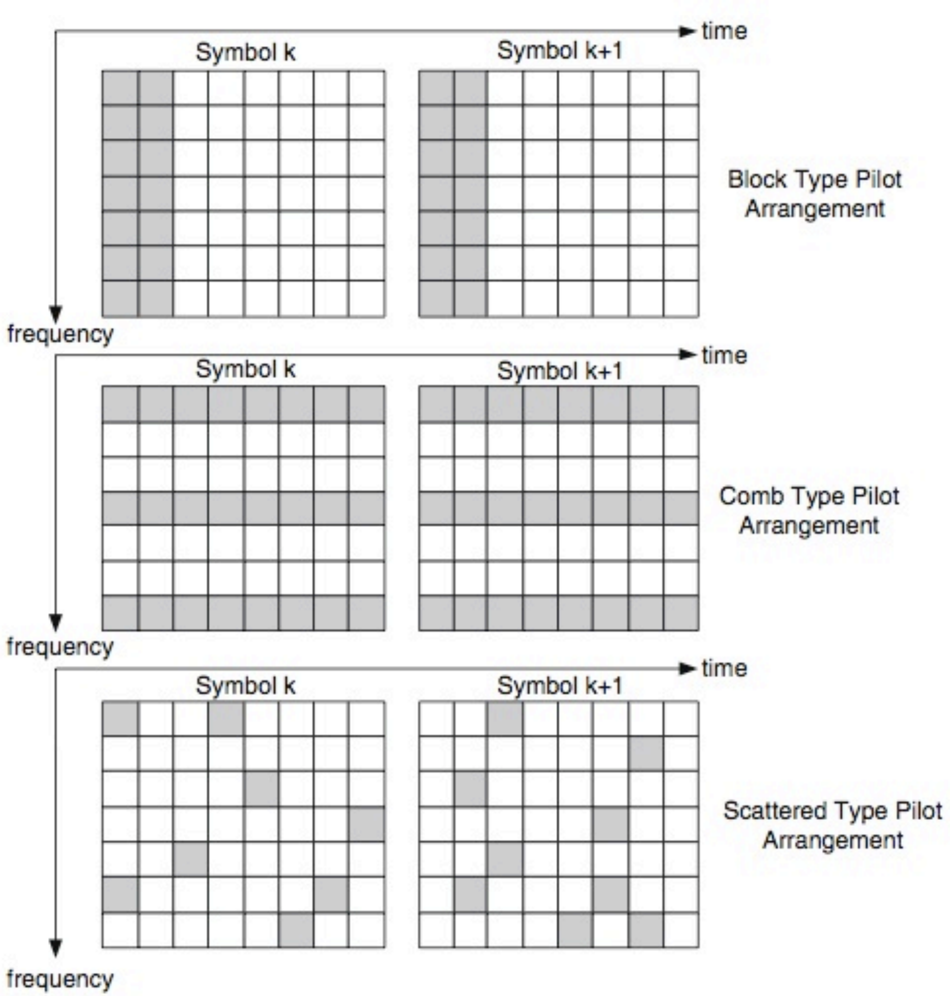


Fig. 4.36 Pilot arrangement for channel estimation

Channel Estimation

The least-squares (LS) channel estimation minimizes ||y-Xh||

$$y = Xh + n$$

$$\hat{h}_{LS} = X^{-1} y$$

$$\hat{h}_{MMSE} = A \hat{h}_{LS}$$

$$A = R_{h\hat{h}_{LS}} R^{-1}_{h\hat{h}_{LS}}$$

Linear minimum mean squared error estimate (LMMSE)

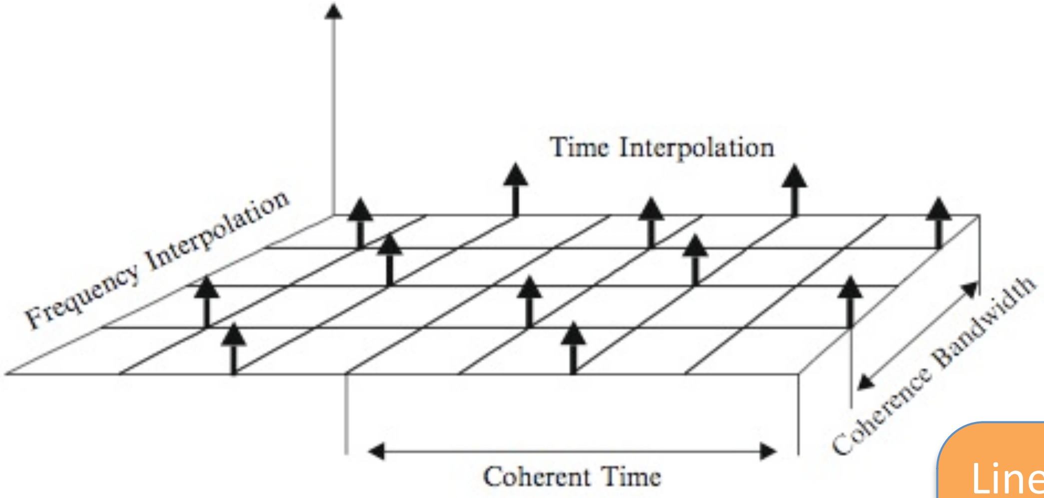


Fig. 4.37 Pilot positioning in time and frequency

Equalization

- Used to combat for intersymbol interference (ISI)

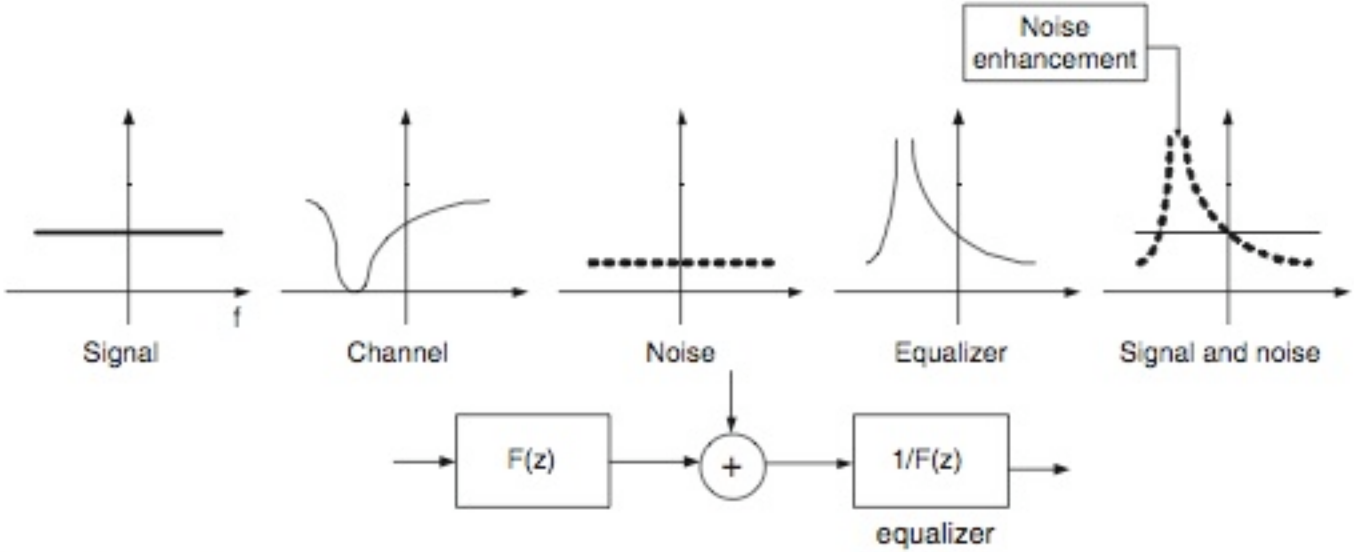


Fig. 4.41 *Zero forcing equalizer*

Equalization in Single Carrier

Complexity of equalizer in single-carrier system increases with data rates, since as data rates increase, the receiver needs to get more frequent samples to compensate for the delay spread and consequently sample clock (t) decreases. These in- crease the number of delay taps in the equalizer and makes it almost impossible to meet rates above 100 Mbps.

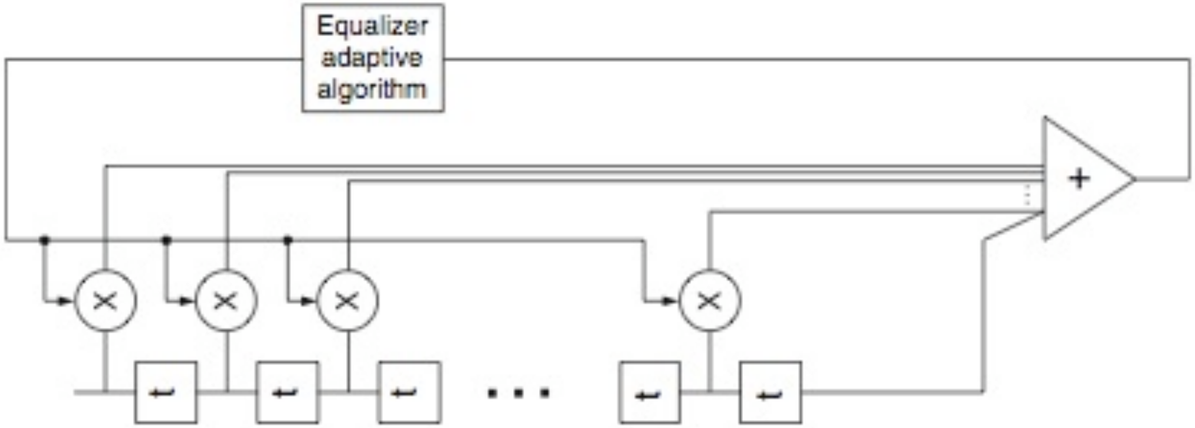


Fig. 5.1 Time domain channel equalizer

MMSE Equalizer

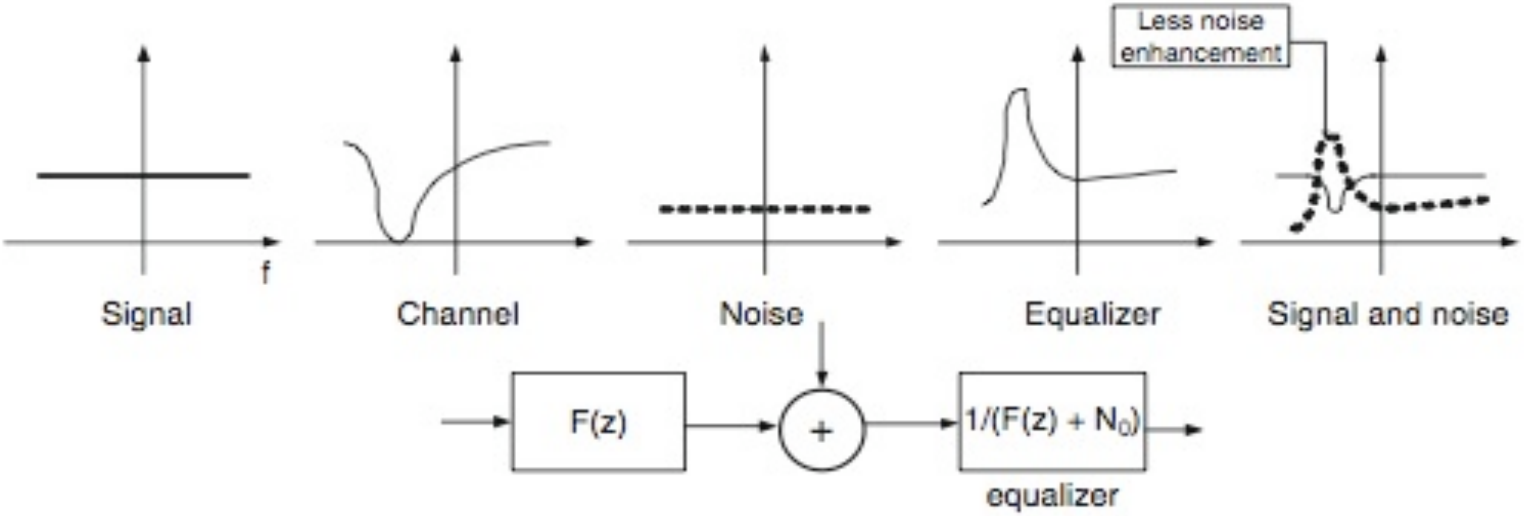


Fig. 4.42 MMSE equalizer

Peak-to-Average Power Ratio and Clipping

- One of the significant drawback of OFDM system is the possibility to experience large peaks since the signal shows a random variable characteristic since it is sum of N independent complex random variables.
- These different carriers may all line up in phase at some instant and consequently produce a high peak, which is quantified by peak-to-average-power ratio (PAPR).

Clipping

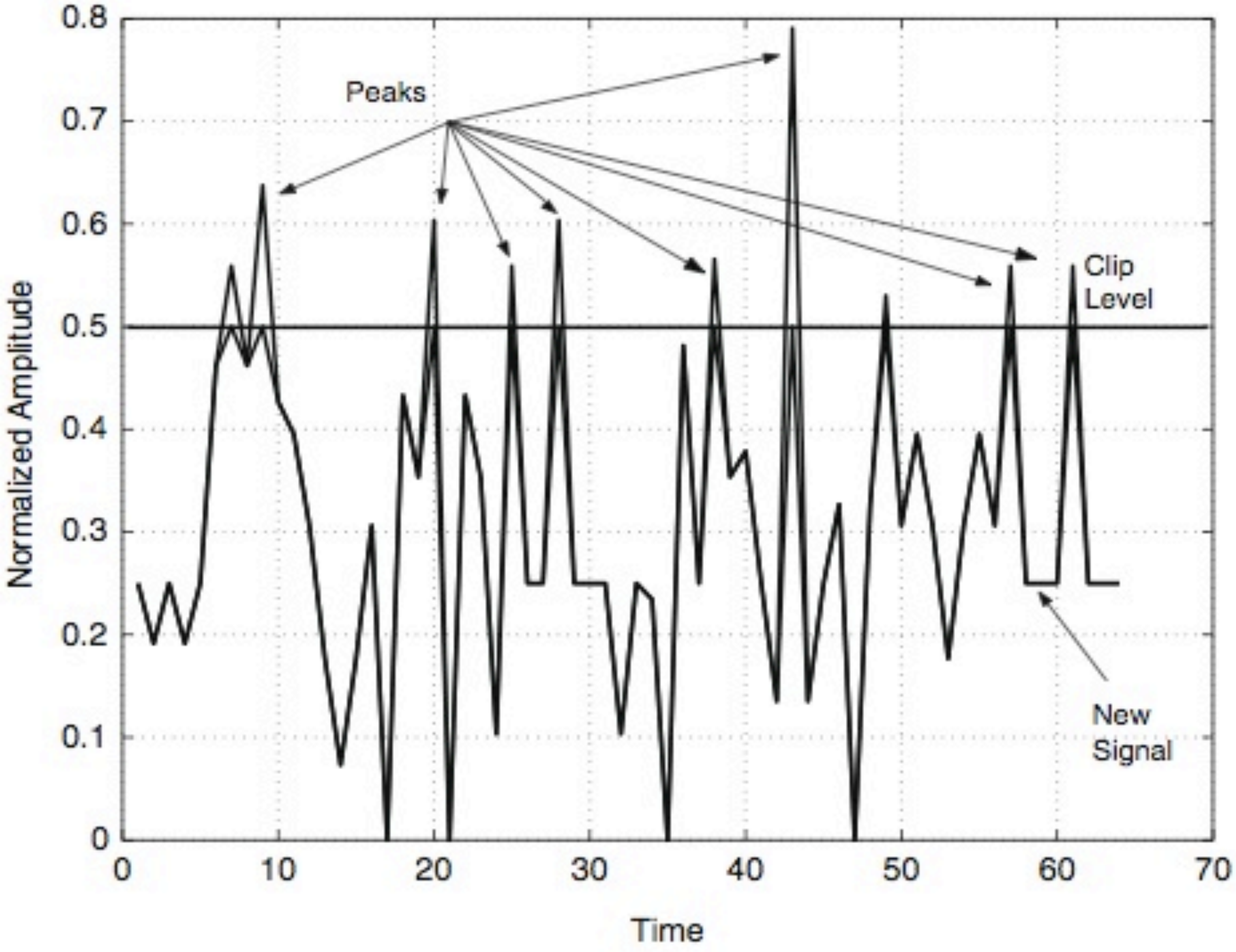


Fig. 4.52 Clipping

Clipping

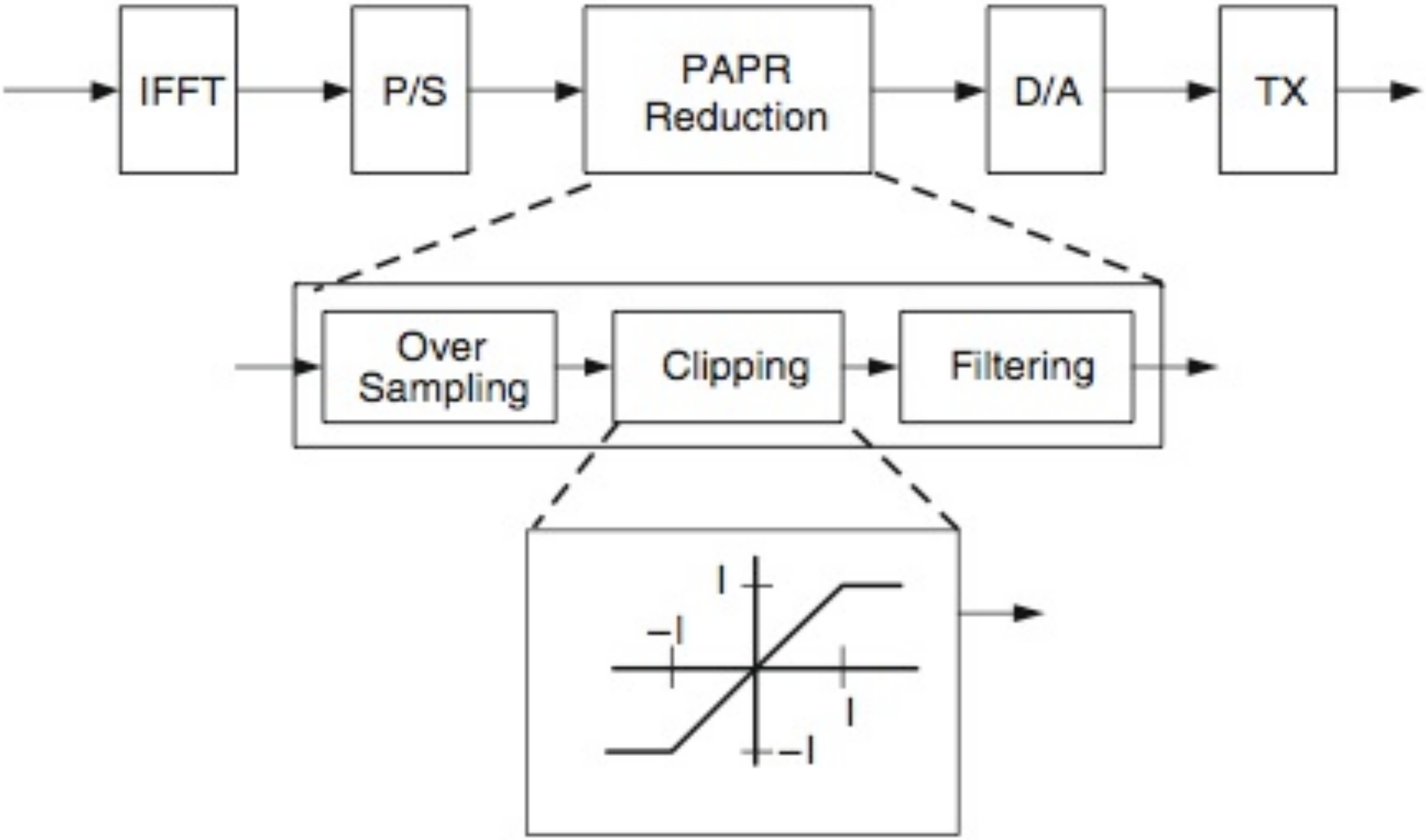


Fig. 4.51 Clipping and Filtering

Clipping

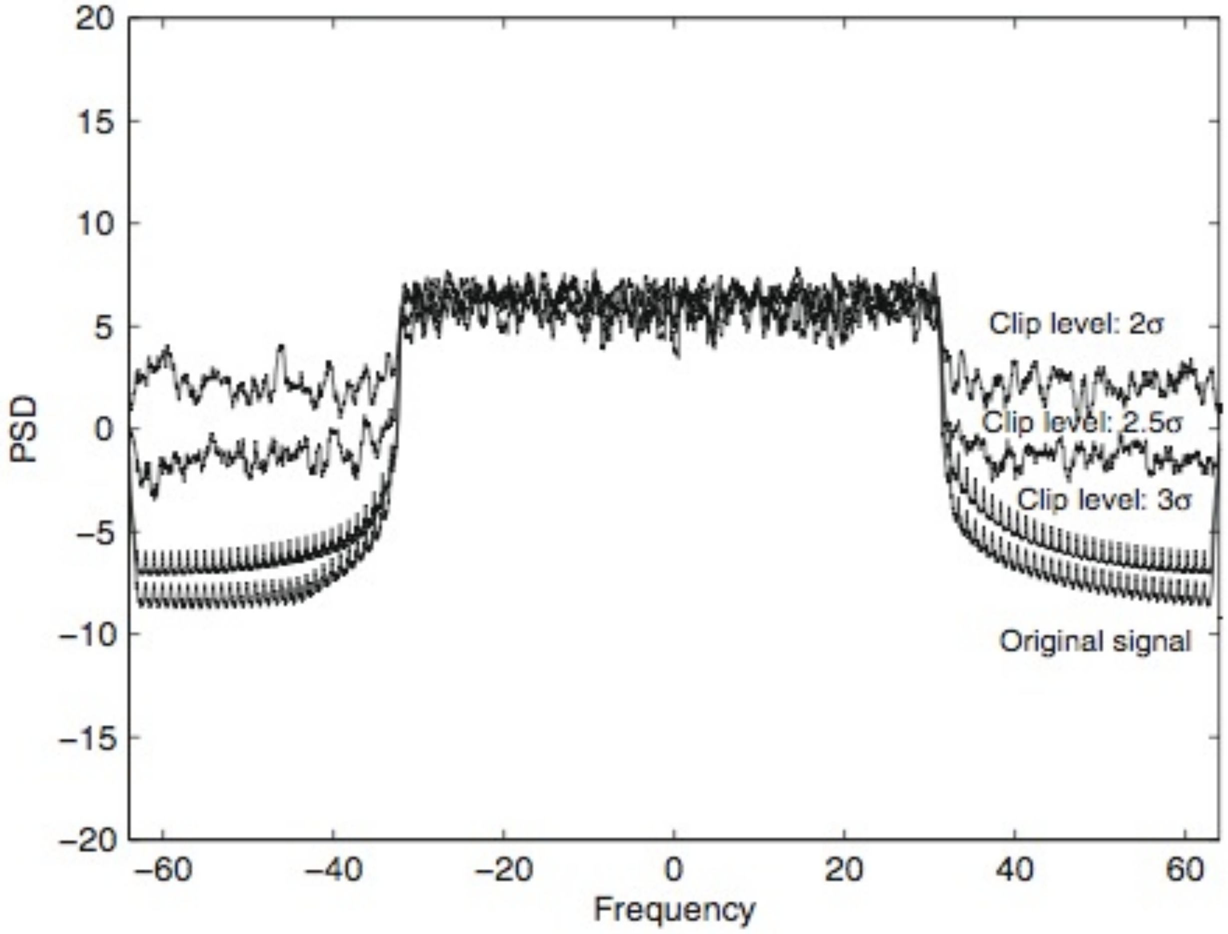


Fig. 4.54 Spectrum

Application: IEEE 802.11a

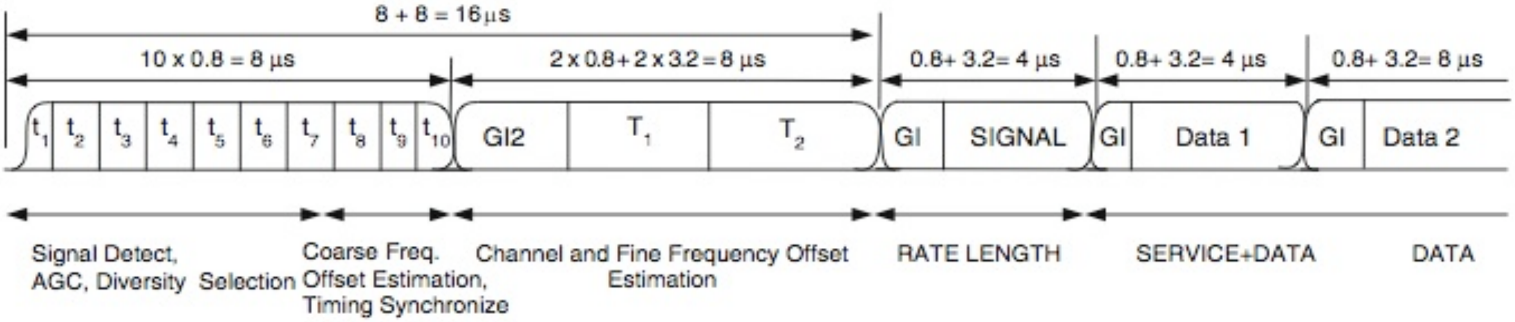
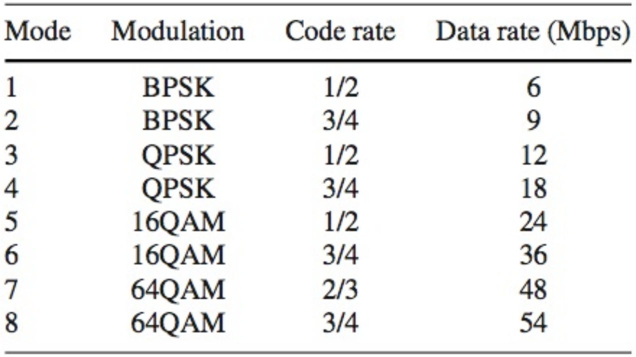


Fig. 4.56 Format of an OFDM frame (© IEEE)



Mode	Modulation	Code rate	Data rate (Mbps)
1	BPSK	1/2	6
2	BPSK	3/4	9
3	QPSK	1/2	12
4	QPSK	3/4	18
5	16QAM	1/2	24
6	16QAM	3/4	36
7	64QAM	2/3	48
8	64QAM	3/4	54

Application: IEEE 802.11a

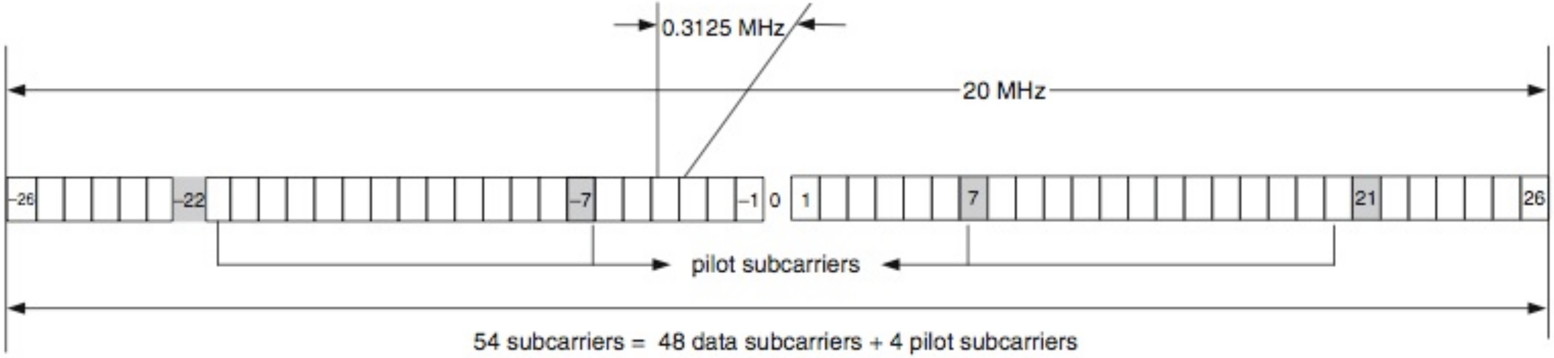


Fig. 4.57 OFDM subcarrier allocation for data and pilot

IEEE 802.11a Parameters	Value
Number of data subcarriers N_{SD}:	48
Number of pilot subcarriers ND_{SP}:	4
Number of subcarriers, total N_{ST}:	52
Subcarrier frequency spacing Δ_F:	0.3125 MHz (=20 MHz/64)
IFFT/FFT period T_{FFT}:	3.2 µs ($1/\Delta_F$)
PLCP preamble duration $T_{PREAMBLE}$:	16 µs ($T_{SHORT} + T_{LONG}$)
Duration of the SIGNAL T_{SIGNAL}:	4.0 µs ($T_{GI} + T_{FFT}$)
GI duration T_{GI}:	0.8 µs ($T_{FFT}/4$)
Training symbol GI duration T_{GI2}:	1.6 µs ($T_{FFT}/2$)
Symbol interval T_{SYM}:	4 µs ($T_{GI} + T_{FFT}$)
Short training sequence duration T_{SHORT}:	8 µs ($10 \times T_{FFT}/4$)
Long training sequence duration T_{LONG}:	8 µs ($T_{GI2} + 2 \times T_{FFT}$)
Signal Bandwidth W:	16.66 MHz

References

- Mobile Broadband by M. Ergen
- OFDM by K. M. Shazad