

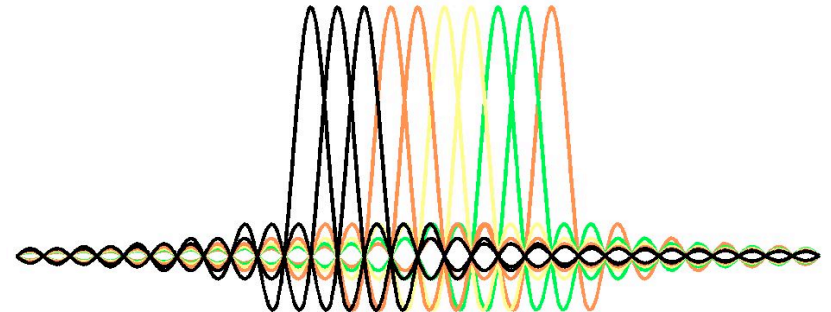


# ELEC 450 & ELEC 550, Introduction to Mobile Broadband

## Lecture 7a: OFDMA Lecture 7b: MIMO

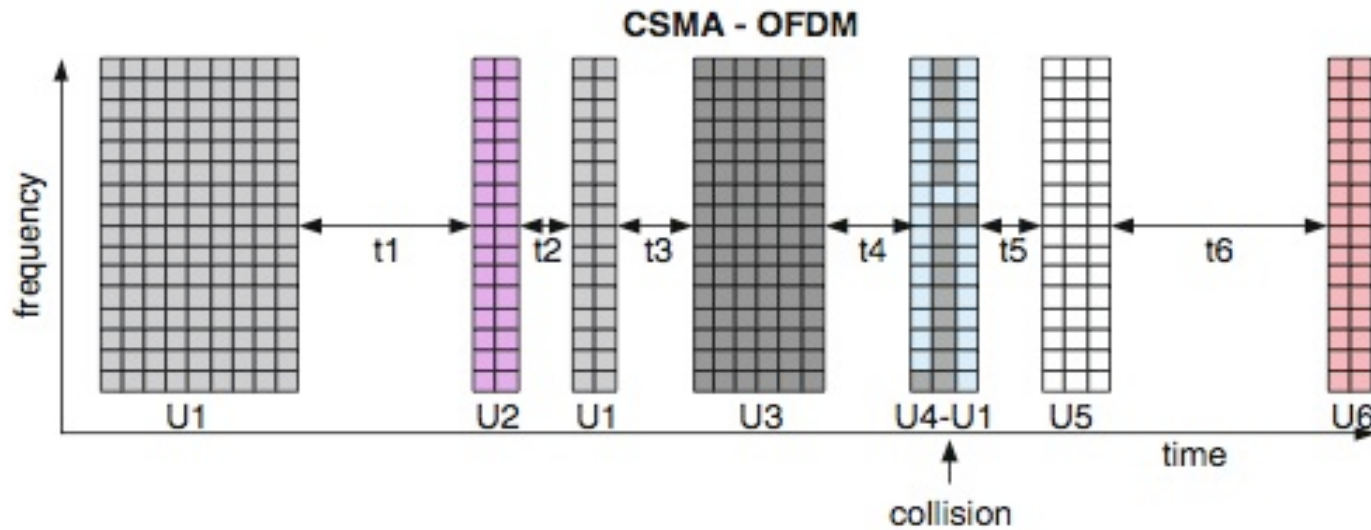
Doc. Dr. Mustafa Ergen  
Fall, 2013  
Koc University

[courses.ku.edu.tr/elec450](http://courses.ku.edu.tr/elec450)

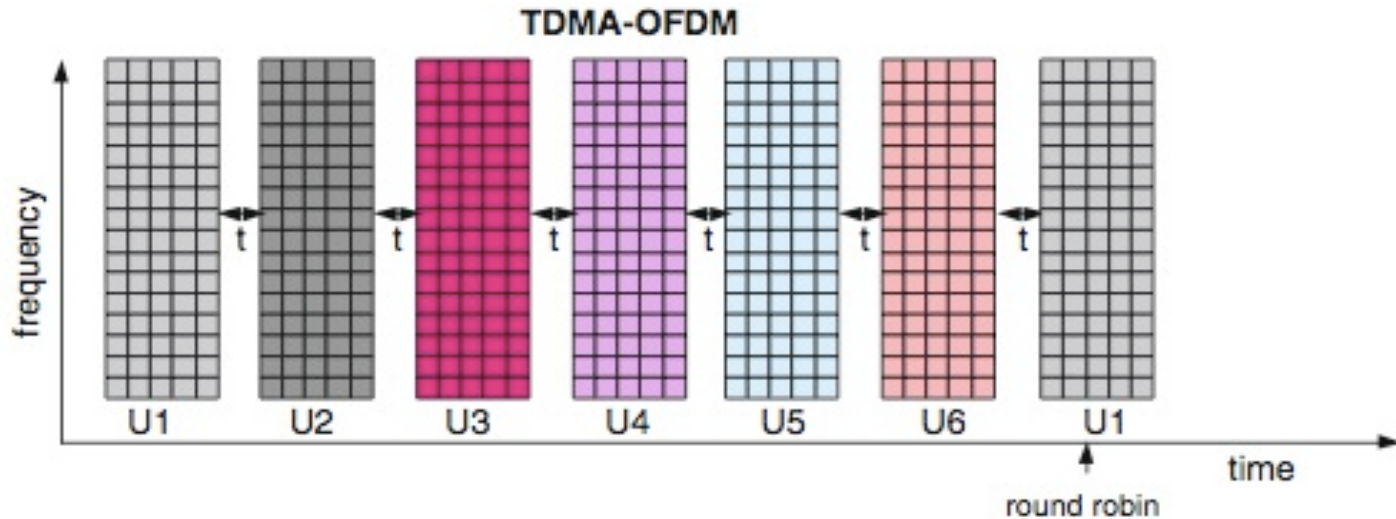




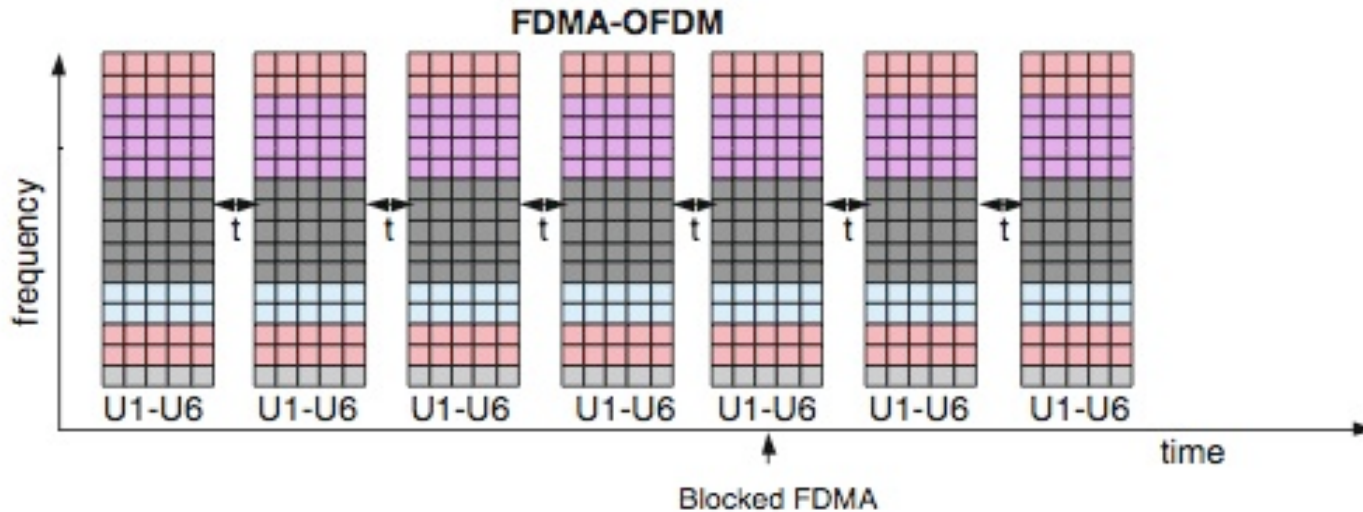
- What is OFDMA?
- Resource Allocation
- Single Frequency Network
- Flash - OFDM



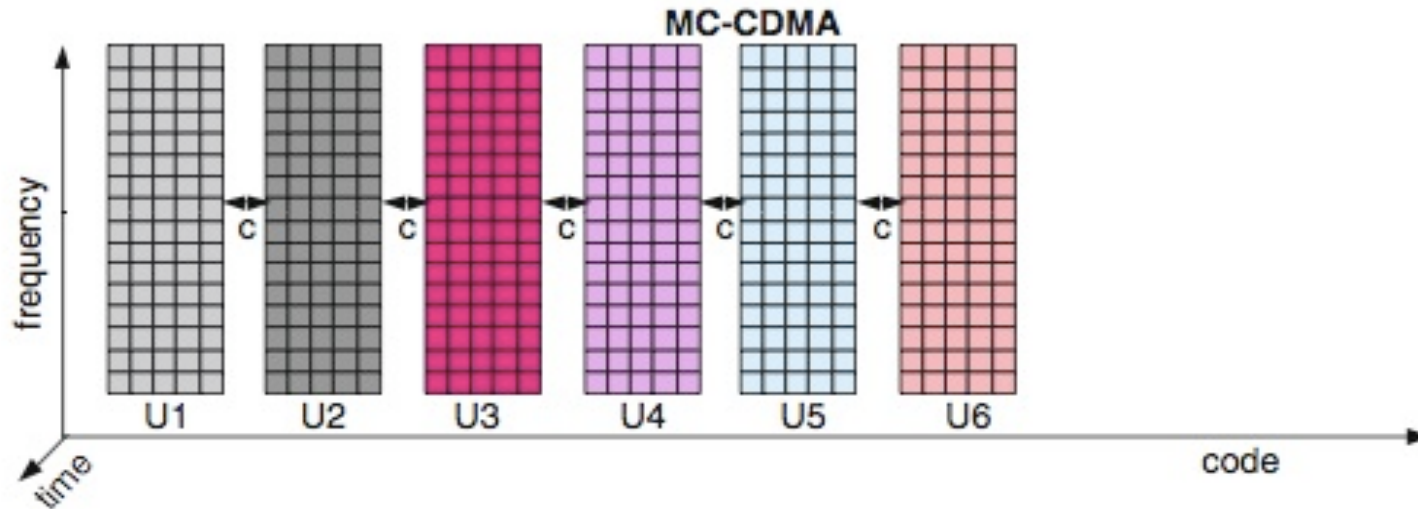
**Fig. 5.2** CSMA-OFDM: There are 6 users (U), and CSMA scheme has random time intervals between frames ( $t_i$ ) and random packet sizes



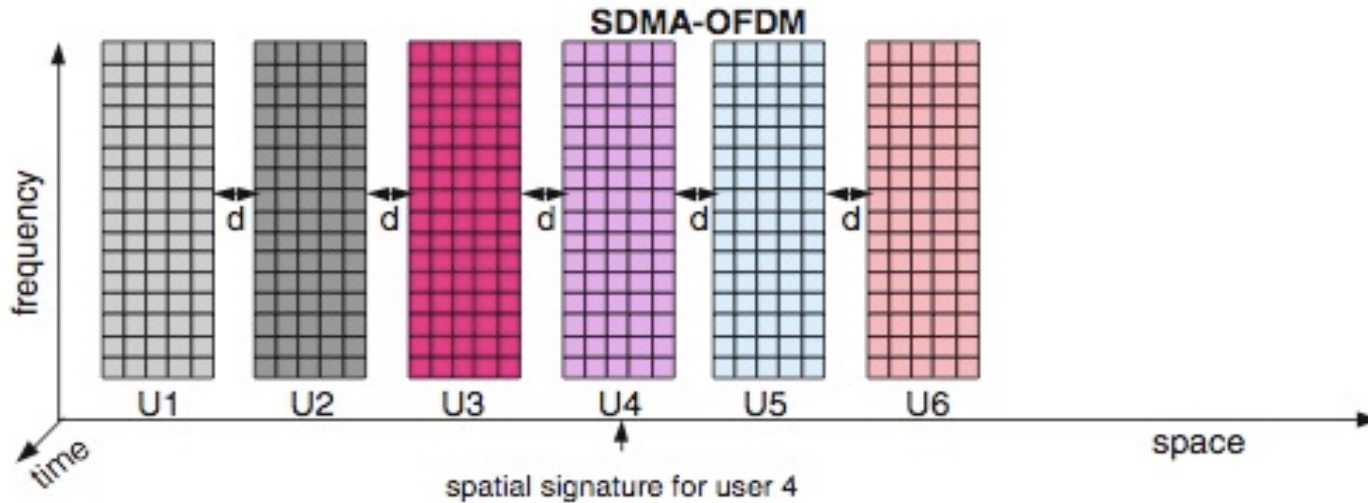
**Fig. 5.3** TDMA-OFDM: There are 6 users (U), and TDMA scheme has fixed time intervals between frames (t) and fixed packet sizes



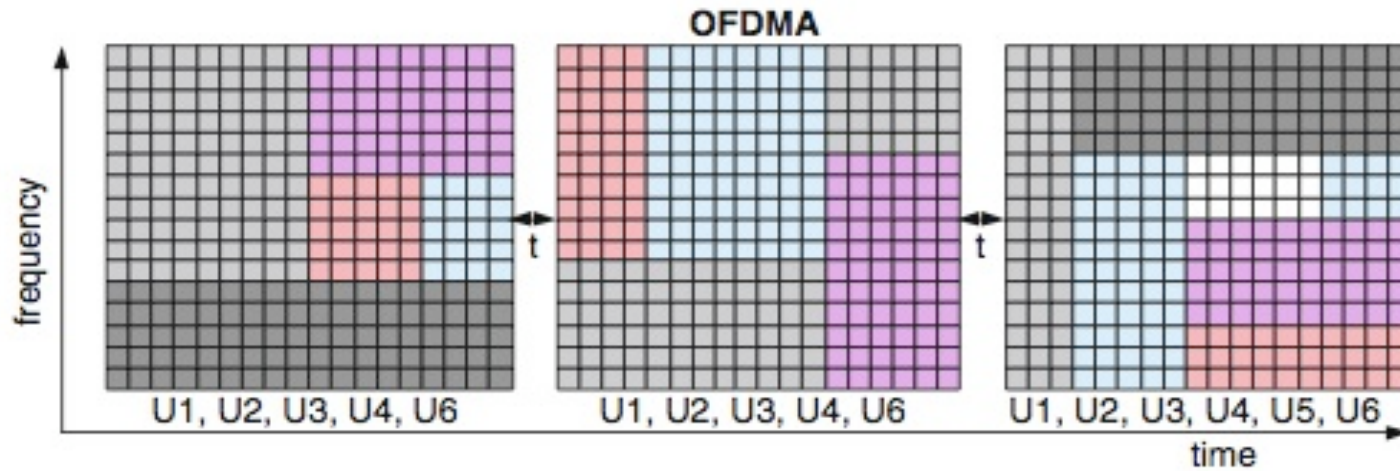
**Fig. 5.4** FDMA-OFDM: There are 6 users (U), and Block-FDMA scheme has fixed time intervals between frames (t) and fixed subcarrier allocation



**Fig. 5.5** MC-CDMA: There are six users (U), and time-spread MC-CDMA scheme has distance between codes (c) for orthogonality

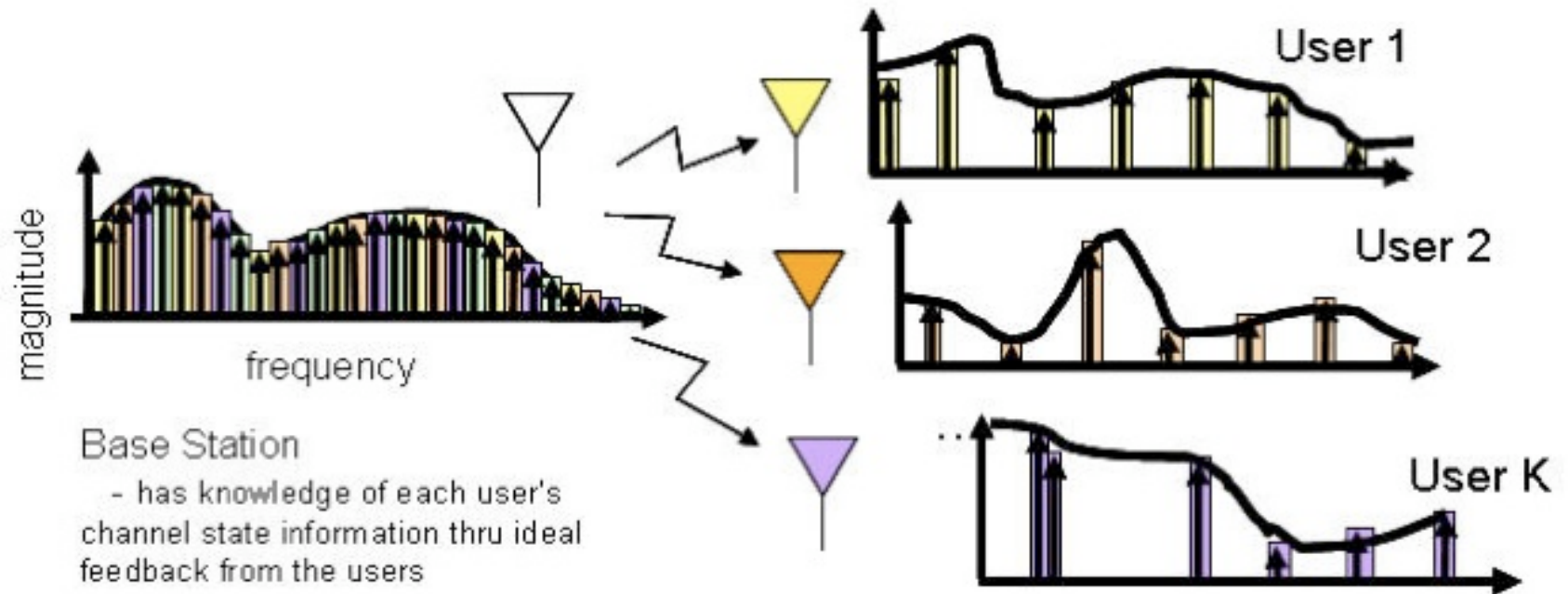


**Fig. 5.6** SDMA-OFDM: There are 6 users (U), and SDMA scheme has physical distance between receivers ( $d$ ) for orthogonality



**Fig. 5.7** OFDMA: There are 6 users (U), and OFDMA scheme has fixed distance between frames (t) and flexible slot and subcarrier allocation

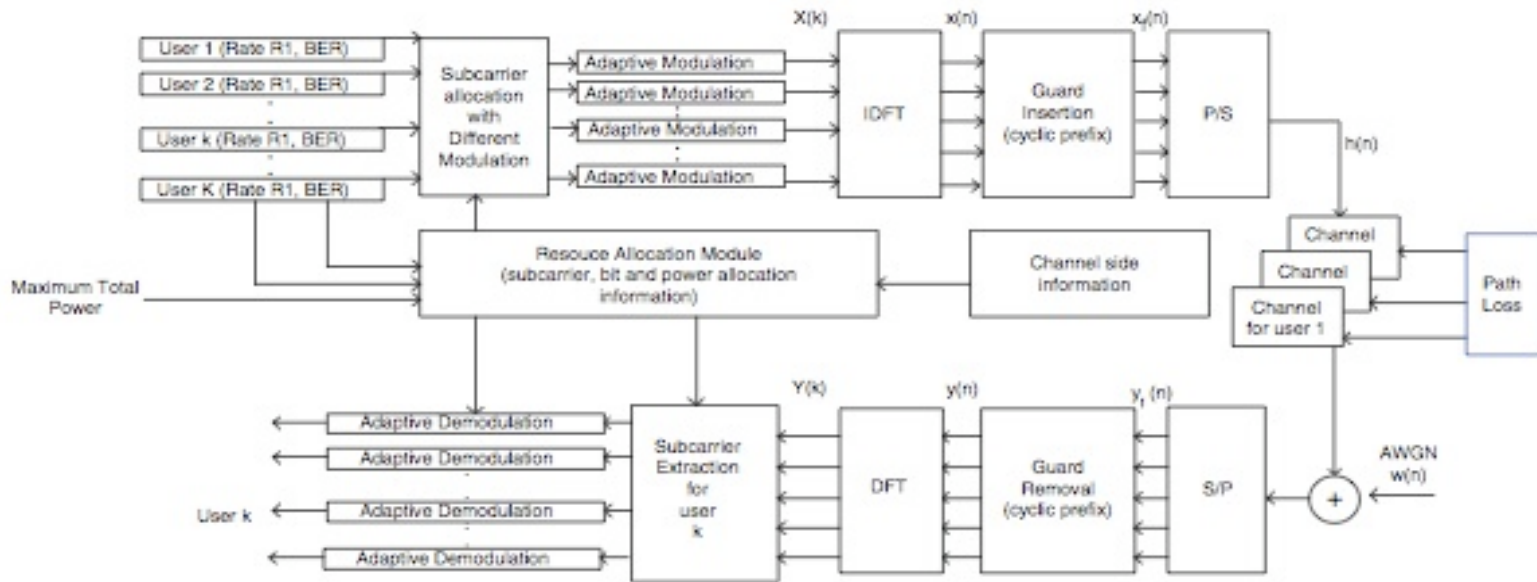






- Power inefficient compared to OFDM
- OFDMA electronics, including FFTs and Forward Error Correction (FEC) is complex.
- Frequency-selective fading and diversity gain benefits could be at least partly lost if very few sub-carriers are assigned to each user, and if the same carrier is used in every OFDM symbol.
- Dealing with co-channel interference from nearby cells is more complex in OFDM than in CDMA.

Source: <http://www.wirelessnetdesignline.com/howto/broadband/198000601;jsessionid=CZFRNRNKHXQGBQE1GHPCKHWATMY32JVN?pgno=2>



**Fig. 5.10** Orthogonal frequency division multiple access system



For example, OFDMA mode of IEEE 802.16e-2005, which is expected to operate between 2–6 GHz, considers mobility at a maximum speed of 125 km/h (35 m/s). Doppler shift for operation in 2.5 GHz carrier frequency ( $f_c$ ) is

$$f_m = \frac{v f_c}{c} = \frac{35 \text{ m/s} \cdot 2.5 \cdot 10^9 \text{ Hz}}{3 \cdot 10^8 \text{ m/s}} = 291 \text{ Hz} \quad (5.1)$$

and it is 408 Hz when  $f_c = 3.5$  GHz and 700 Hz when  $f_c = 6$  GHz. When maximum Doppler shift is considered, coherence time of the channel is  $T_C = \sqrt{\frac{9}{16 \cdot \pi \cdot f_m^2}} = 1.1$  ms. This would require an update rate of  $\approx 1$  KHz for channel estimation and equalization. The delay spread ( $T_S$ ) for mobile environment is  $20 \mu\text{s}$  specified by The International Telecommunications Union (ITU-R).<sup>2</sup> Associated coherence bandwidth ( $B_C$ ) is

$$B_C \approx \frac{1}{5 \cdot T_S} = \frac{1}{5 \cdot 20 \mu\text{s}} = 10 \text{ KHz}, \quad (5.2)$$

where sought frequency correlation is 50%. This means that over a 10-KHz subcarrier width, the fading is considered flat.

SOFDMA subcarrier spacing is independent of channel bandwidth. Scalability ensures that system performance is consistent across different RF channel sizes (1.25–20 MHz).

- With the channel information, the objective of resource allocation problem can be defined as maximizing the throughput subject to a given total power constraint regarding the user's QoS requirements.

C is bits per symbol

Indicator: 1 if that subcarrier is allocated

$$\max_{c_{k,n}, \gamma_{k,n}} R_k = \sum_{n=1}^N c_{k,n} \gamma_{k,n} \quad \text{for all } k$$

$$\text{subject to } P_T = \sum_{k=1}^K \sum_{n=1}^N \frac{f_k(c_{k,n}, \text{BER}_k)}{\alpha_{k,n}^2} \gamma_{k,n} \leq P_{\max},$$

Power allocated to the nth user

# Subcarrier Allocation: Fixed QoS

User	1	2	3	4
Rate (bits)	12	6	6	8
BER	1e-2	1e-2	1e-4	1e-4

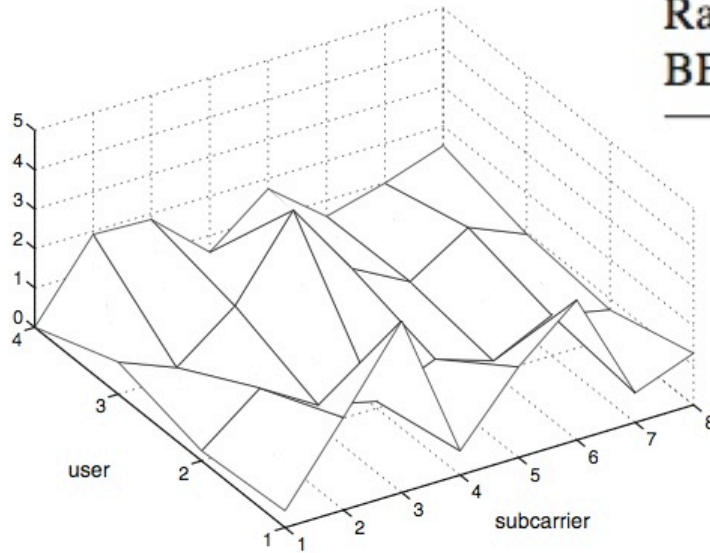


Fig. 5.8 A time instance of wireless channel for each user

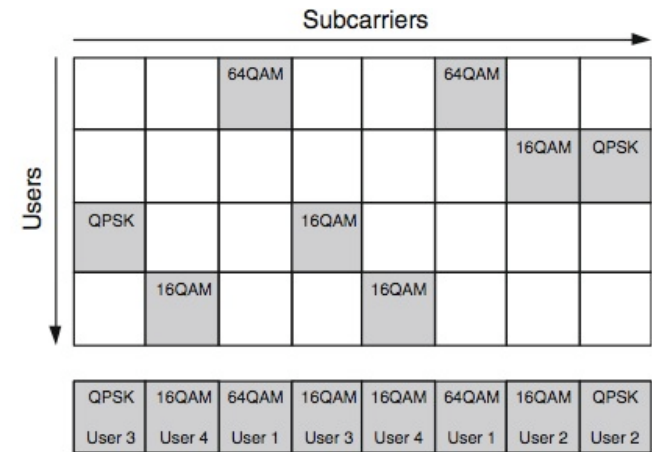
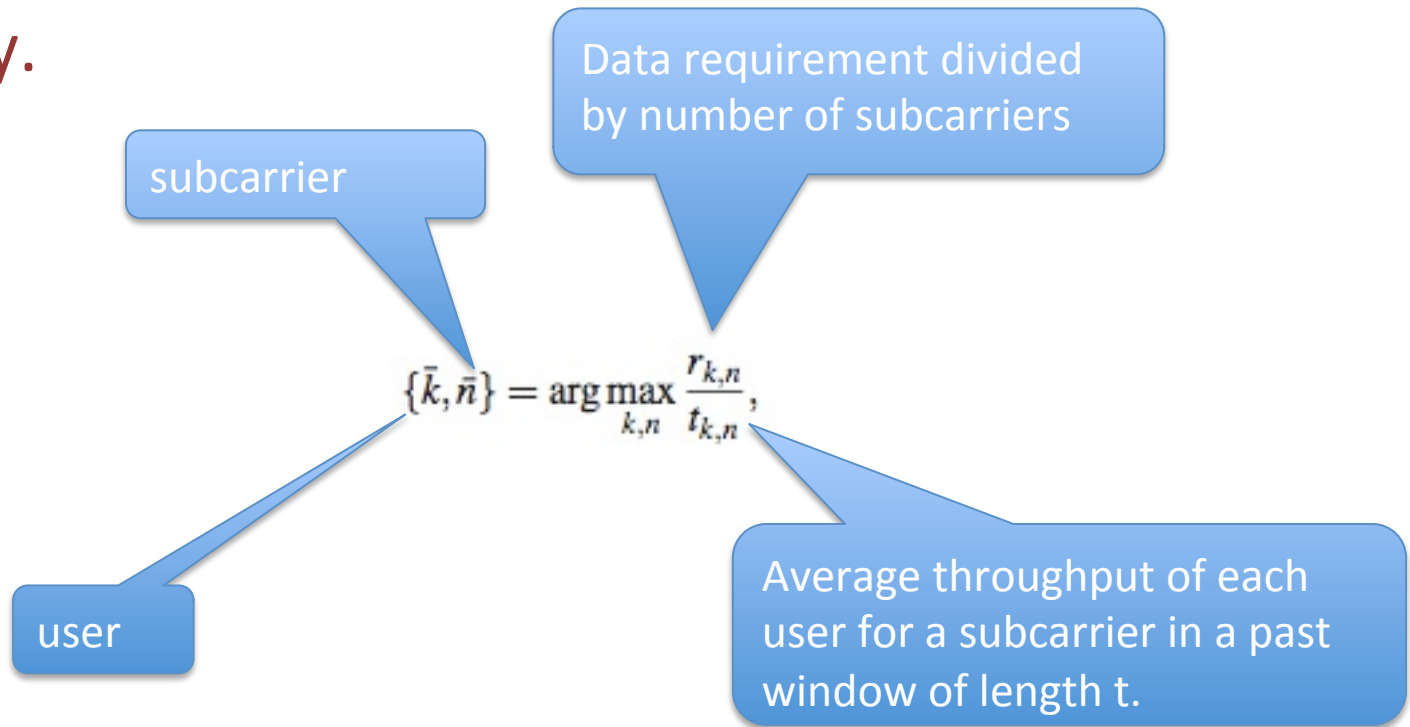


Fig. 5.9 Optimal resource allocation and bit loading



## Subcarrier Allocation: Variable QoS

- Another way to approach resource allocation is in terms of capacity. Suppose there is no fixed requirements per symbol and the aim is to maximize capacity.



# Single Frequency Network

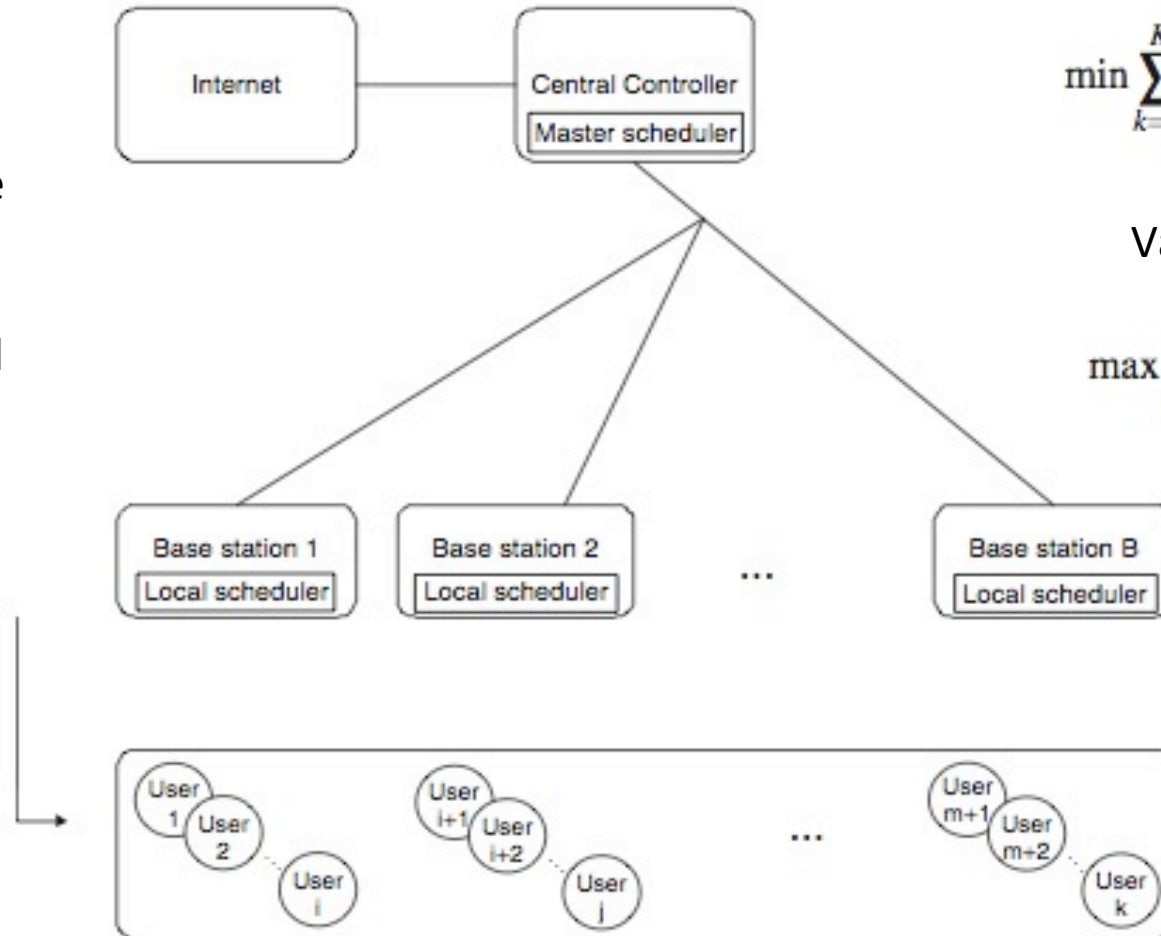
Fixed QoS

$$\min \sum_{k=1}^K \sum_{n=1}^N \sum_{b=1}^B \gamma_{k,n,b} P_{k,n,b}$$

Variable QoS

$$\max \sum_{k=1}^K \sum_{n=1}^N \sum_{b=1}^B \gamma_{k,n,b} C_{k,n,b}$$

Removes the requirement for cell planning and called single frequency network



**Fig. 5.20** Cellular OFDMA architecture



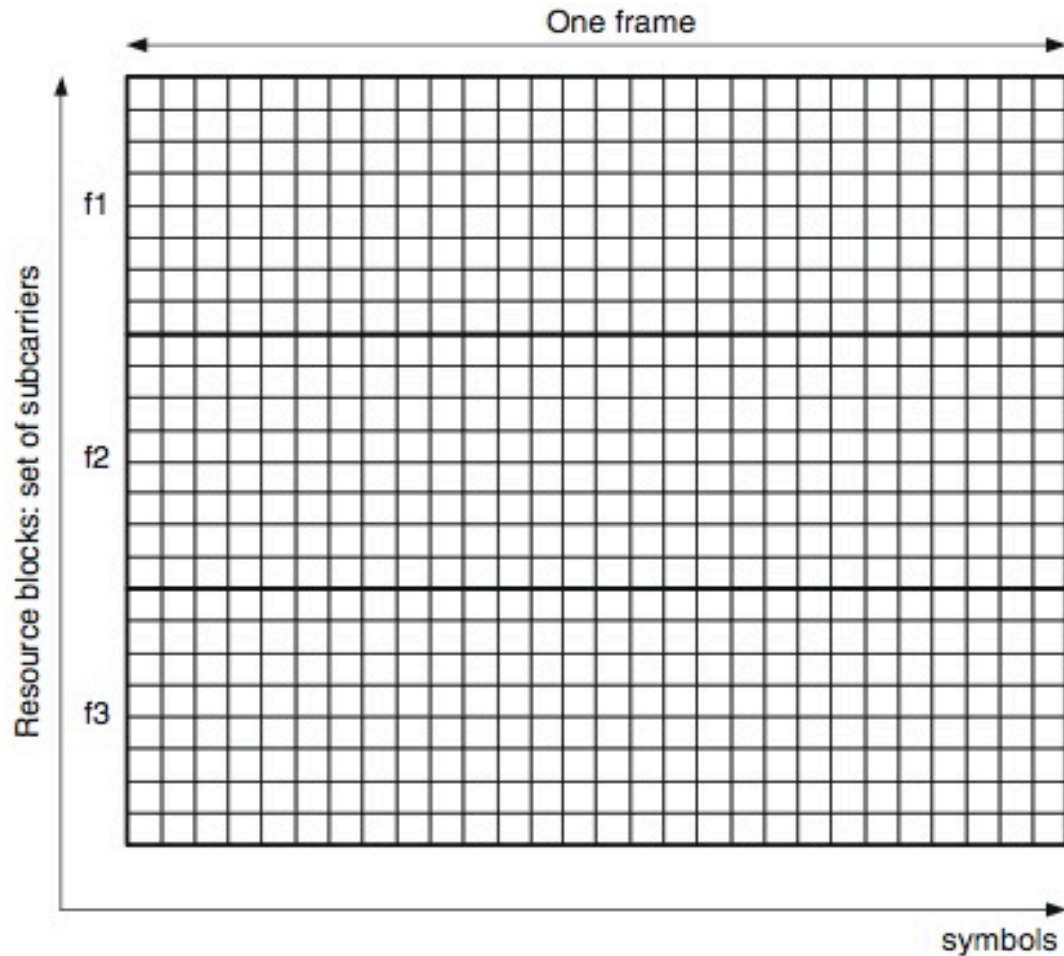


Fig. 5.22 Segmentation of an OFDMA frame

# SFN: Frame Allocation

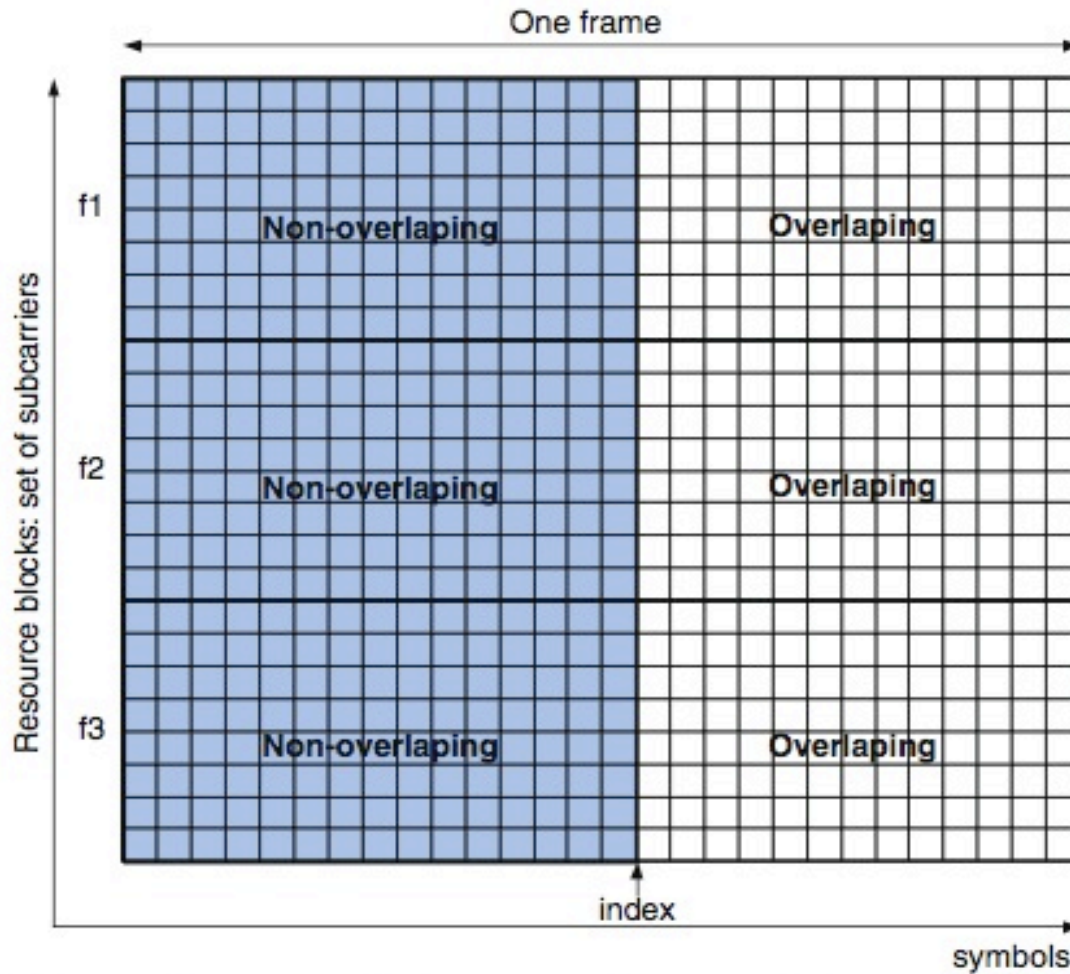
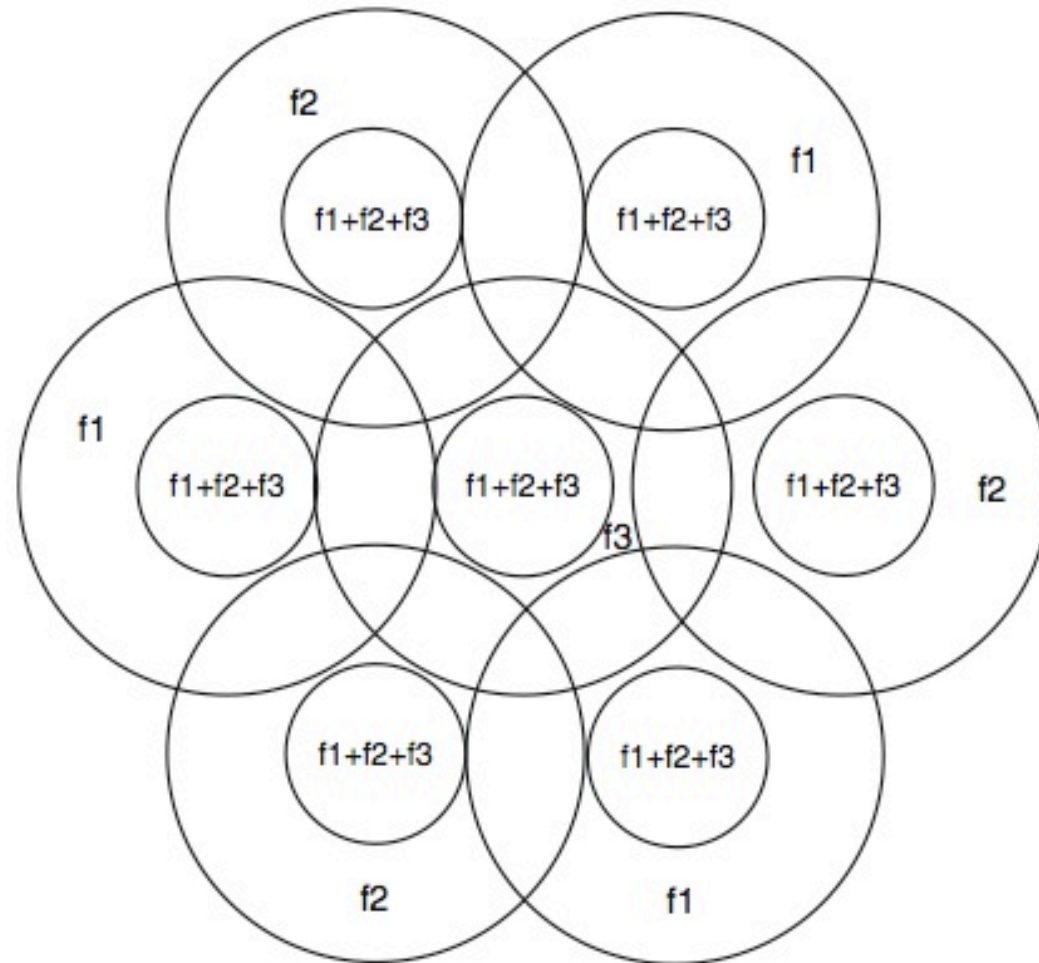
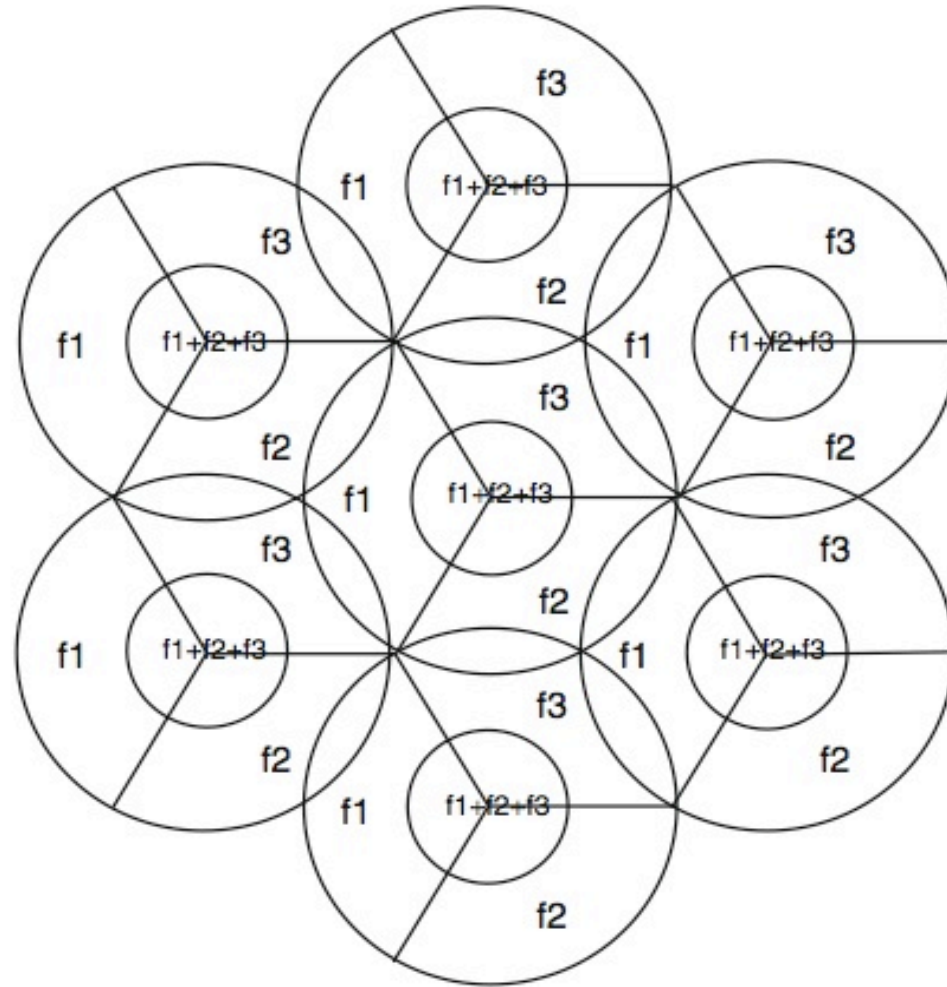


Fig. 5.25 Segmentation of an OFDMA frame for partial reuse without transmit power control

# SFN: Omni Directional



**Fig. 5.23** Full reuse with reduced coverage in one-sector base station

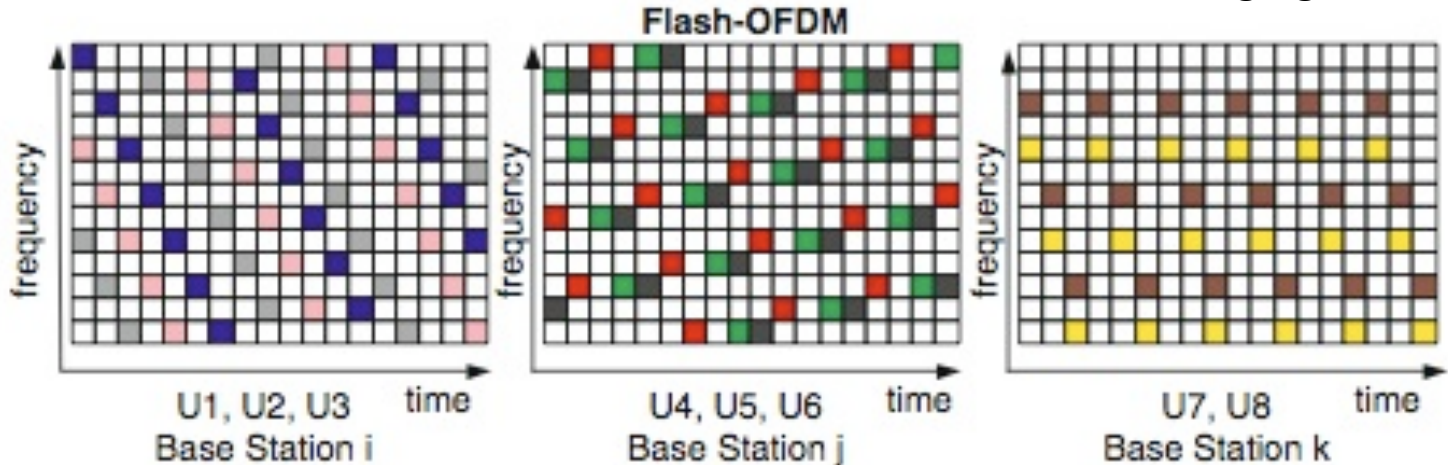


**Fig. 5.24** Full reuse with power control in three-sector base station



# Flash OFDM

IEEE 802.20: Besides OFDMA, CDMA is used for interference averaging.



**Fig. 5.26** Code-based hopping pattern for Flash-OFDM

Flash-OFDM utilizes an hopping pattern in time and frequency in order to exploit interference diversity as in CDMA as well as frequency diversity. Interference diversity averages out the interference coming from more user, since interference coming solely from one user causes severe interference to each other. A hopping pattern is assigned to a user, which alternates the subcarriers at each symbol.

Periodic hopping patterns consider frequency diversity and interference diversity. Frequency diversity is exploited by allocating subcarriers as spread as possible and alternate them every symbol time. Interference diversity also considers to allocate hop patterns that are as “apart” as possible from adjacent base stations. If there are  $N$  subcarriers then  $N$  can be selected as the hopping period and there can be  $N$  channels.

## Latin Square

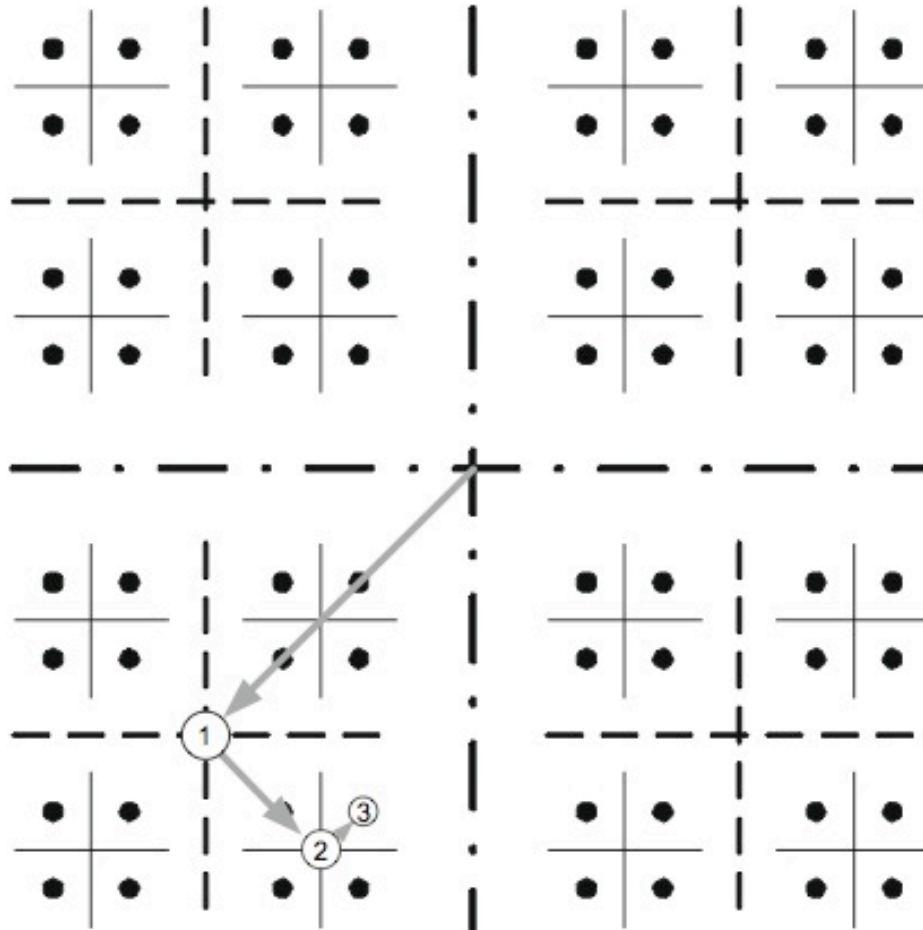
$$[1] \quad \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \\ 3 & 5 & 4 & 2 & 1 \\ 4 & 1 & 5 & 3 & 2 \\ 5 & 3 & 2 & 1 & 4 \end{bmatrix}$$

## No. of Latin squares

$N$	No. of Latin squares
1	1
2	2
3	12
4	576
5	161280
6	812851200
7	61479419904000
8	108776032459082956800
9	5524751496156892842531225600
10	9982437658213039871725064756920320000
11	776966836171770144107444346734230682311065600000

# Subcarrier Sharing: Embedded Modulation

Each carrier carries 6 bits for 64QAM modulation and each user uses only 2 out of 6.



This scheme allows a subcarrier to be used by more than one user. This scheme is known as embedded modulation and exploits the fact that a subcarrier that is high quality to a user maybe high quality to multiple users and therefore that subcarrier is utilized to carry bits of multiple users.

**Fig. 5.31** Embedded modulation for 64QAM: Three 4-QAM modulation is embedded to address three different users



- Mobile Broadband by M. Ergen





# ELEC 450 & ELEC 550, Introduction to Mobile Broadband

## Lecture 7b: MIMO

Doc. Dr. Mustafa Ergen  
Fall, 2013  
Koc University

[courses.ku.edu.tr/elect](http://courses.ku.edu.tr/elect)

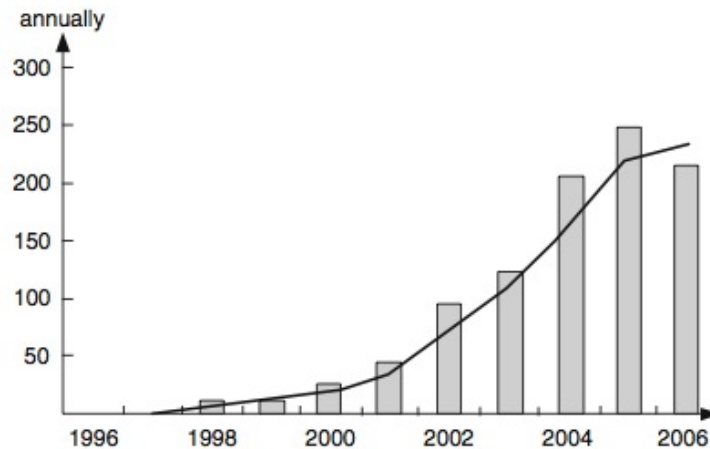


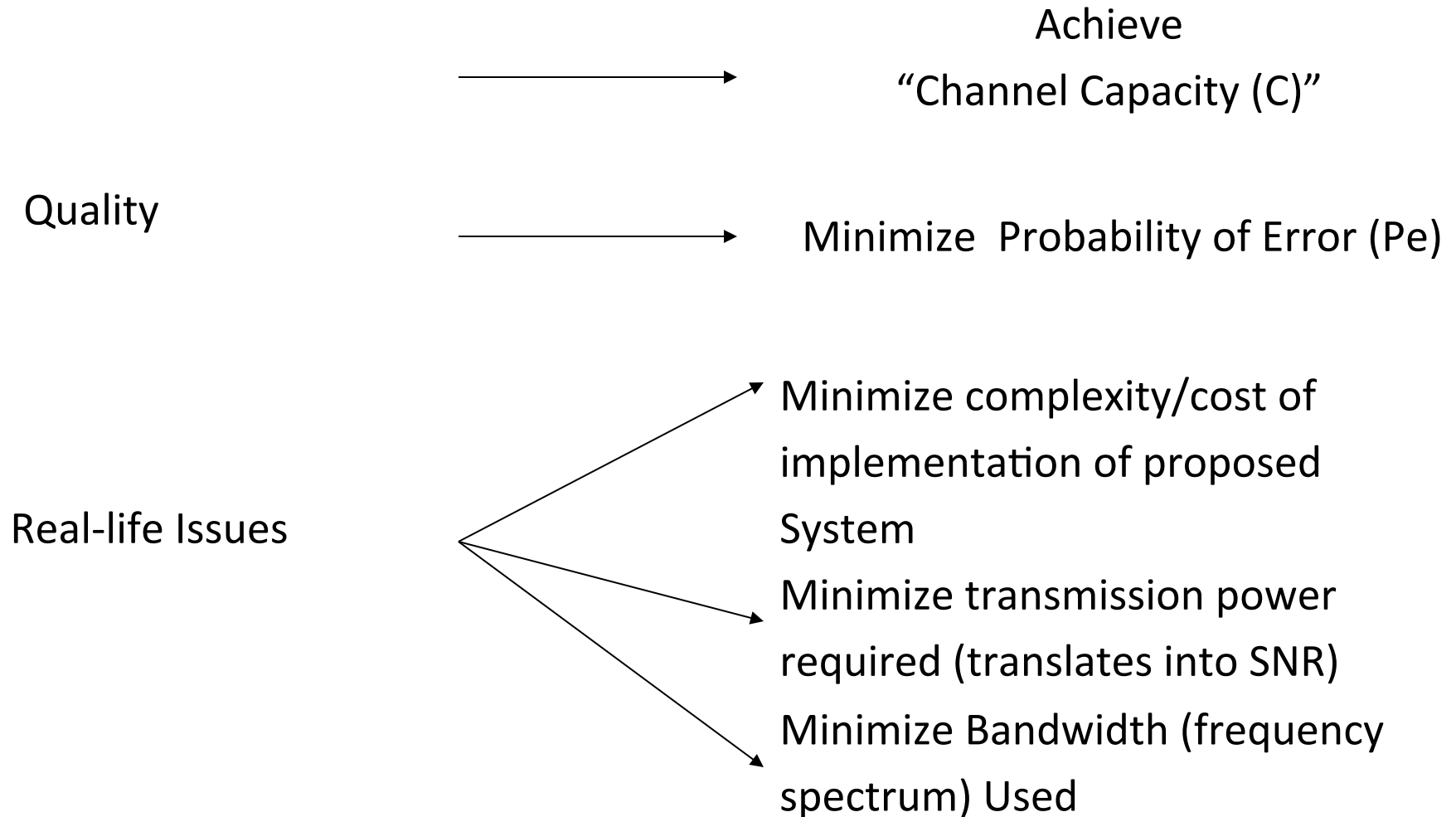
Fig. 6.1 MIMO patent applications per year. (Source: Marvedis)



- Introduction...why MIMO??
- Shannon capacity of MIMO systems
- The "pipe" interpretation **Telatar, AT&T 1995**
- To exploit the MIMO channel
  - BLAST
  - Space Time Coding **Foschini, Bell Labs 1996**
  - Beamforming **Tarokh, Seshadri & Calderbank 1998**
- Comparisons & hardware issues
- Space time coding in 3G & EDGE **Release '99**

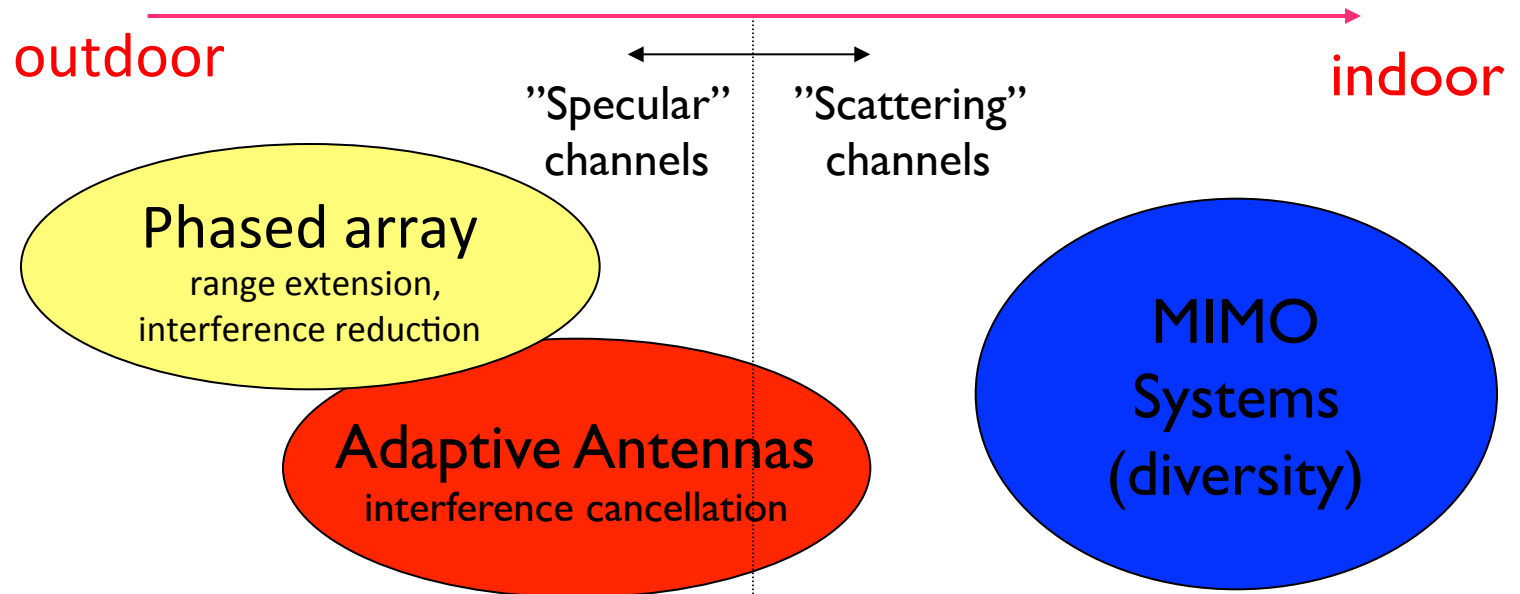
# Aspirations (Mathematical) of a System Designer

- High data rate



# Why MIMO?

- Frequency and time processing are at limits
- Space processing is interesting because it does not increase bandwidth



- **Single-Input-Single-Output (SISO) antenna system**



- Theoretically, the 1Gbps barrier can be achieved using this configuration if you are allowed to use much power and as much BW as you so please!
- Extensive research has been done on SISO under power and BW constraints. A combination a smart modulation, coding and multiplexing techniques have yielded good results but far from the 1Gbps barrier



# MIMO Antenna Configuration

- Use multiple transmit and multiple receive antennas for a single user



- Now this system promises enormous data rates!



# Shannon's Capacity (C)

- Given a unit of BW (Hz), the max error-free transmission rate is
- $C = \log_2(1+SNR)$  bits/s/Hz
- Define
- R: data rate (bits/symbol)
- RS: symbol rate (symbols/second)
- w: allotted BW (Hz)
- Spectral Efficiency is defined as the number of bits transmitted per second per Hz
- $R \times RS$  bits/s/Hz
- $W$
- As a result of filtering/signal reconstruction requirements,  $RS \leq W$ . Hence Spectral Efficiency = R if  $RS = W$
- If I transmit data at a rate of  $R \leq C$ , I can achieve an arbitrarily low  $P_e$

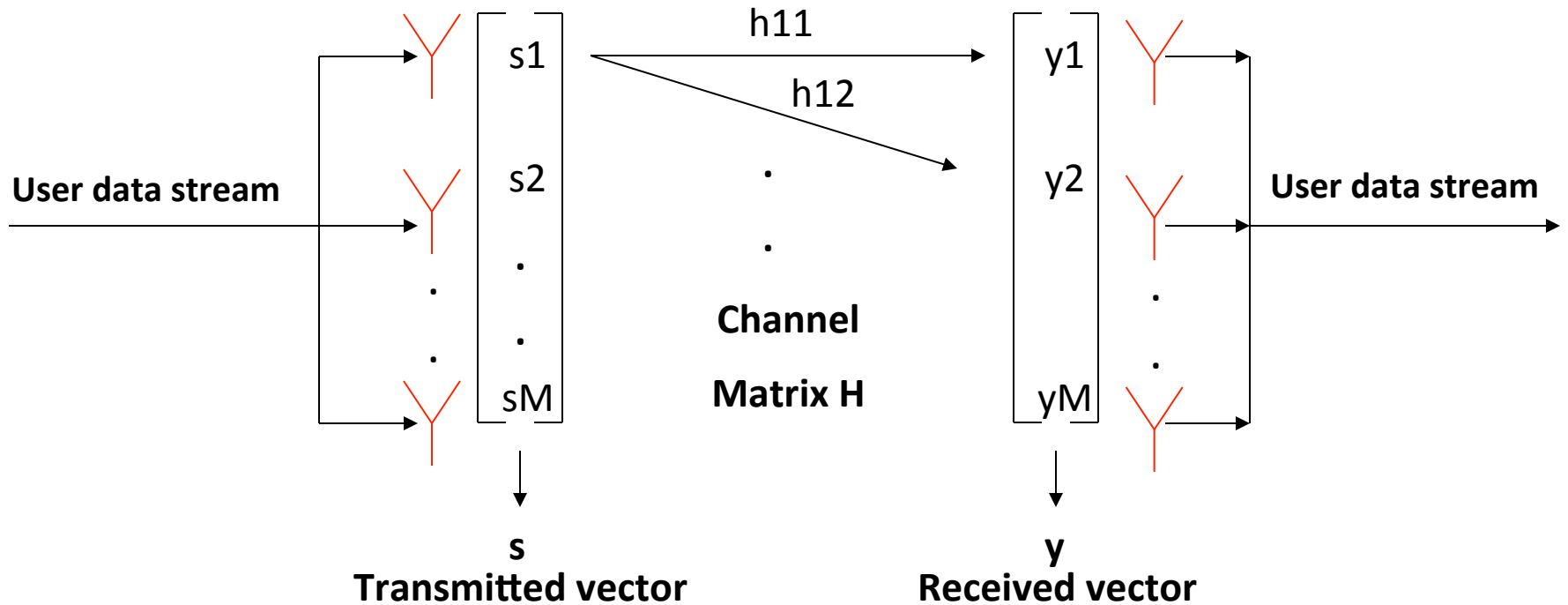


Scheme	b/s/Hz
BPSK	1
QPSK	2
16-QAM	4
64-QAM	6

- Spectral efficiencies of some widely used modulation schemes
- The Whole point: Given an acceptable  $P_e$ , realistic power and BW limits, MIMO Systems using smart modulation schemes provide much higher spectral efficiencies than traditional SISO



# MIMO System Model



$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

Where  $\mathbf{H} =$

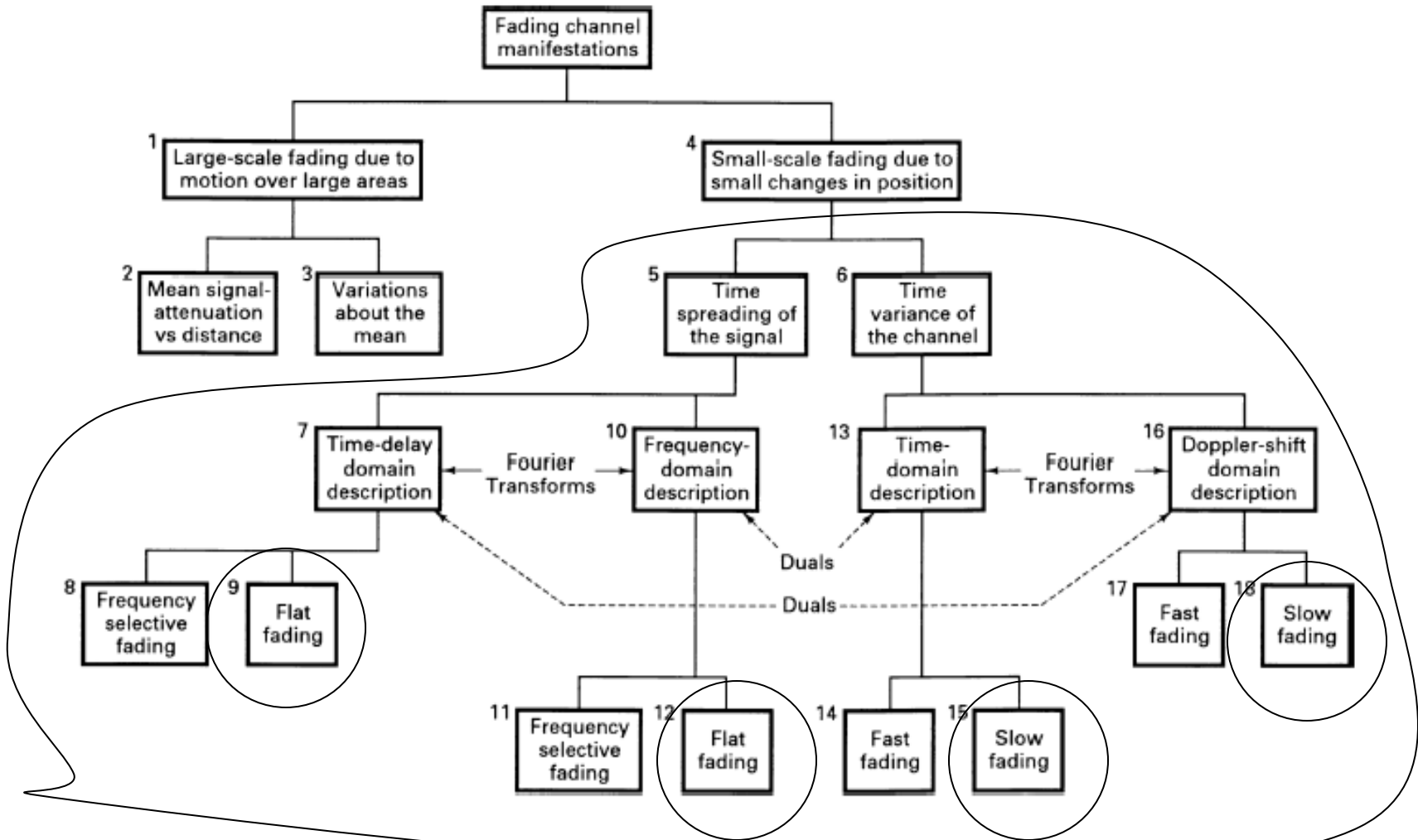
$$\begin{array}{c}
 \uparrow \\
 \text{MR} \\
 \left[ \begin{array}{cccc}
 h_{11} & h_{21} & \dots & h_{M1} \\
 h_{12} & h_{22} & \dots & h_{M2} \\
 \cdot & \cdot & \dots & \cdot \\
 h_{1M} & h_{2M} & \dots & h_{MM}
 \end{array} \right] \\
 \downarrow
 \end{array}$$

← MT →

$h_{ij}$  is a Complex Gaussian random variable that models fading gain between the  $i$ th transmit and  $j$ th receive antenna



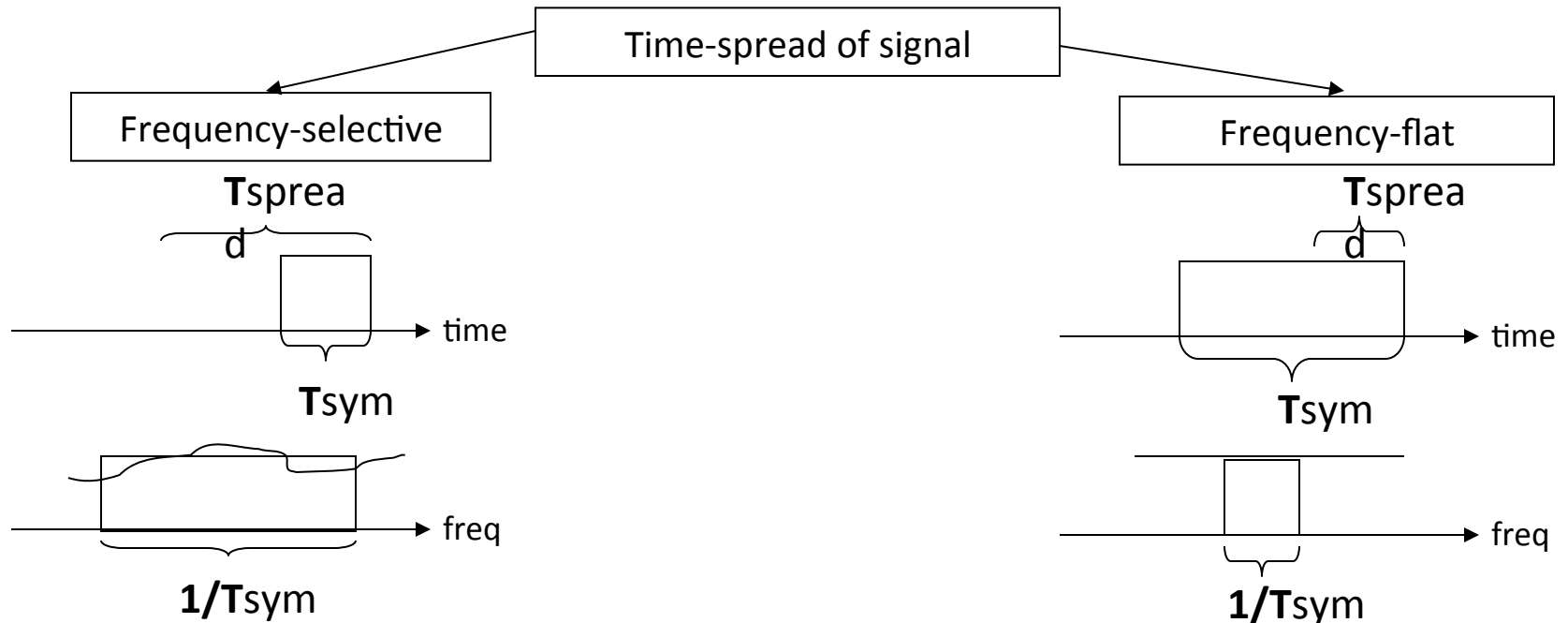
# Types of Channels





# Fading Channels

- *Fading* refers to changes in signal amplitude and phase caused by the channel as it makes its way to the receiver
- Define  $T_{\text{spread}}$  to be the time at which the last reflection arrives and  $T_{\text{sym}}$  to be the symbol time period



Occurs for wideband signals (small  $T_{\text{sym}}$ )

Occurs for narrowband signals (large  $T_{\text{sym}}$ )

**TOUGH TO DEAL IT!**

**EASIER! Fading gain is complex Gaussian**

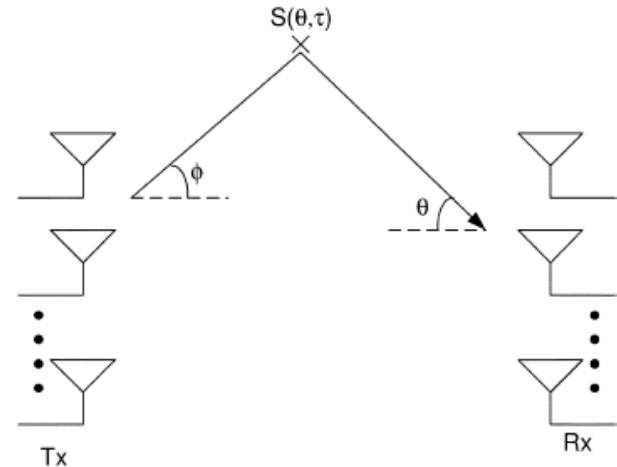
**Multipaths NOT resolvable**

- In addition, assume slow fading
- MIMO Channel Response

$$\mathbf{H}(\tau, t) = \begin{bmatrix} h_{1,1}(\tau, t) & h_{1,2}(\tau, t) & \cdots & h_{1,M_T}(\tau, t) \\ h_{2,1}(\tau, t) & h_{2,2}(\tau, t) & \cdots & h_{2,M_T}(\tau, t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_R,1}(\tau, t) & h_{M_R,2}(\tau, t) & \cdots & h_{M_R,M_T}(\tau, t) \end{bmatrix}$$

Channel Time-variance

Time-spread



- Taking into account slow fading, the MIMO channel impulse response is constructed as,

$$\mathbf{H}(\tau) = \int_{-\pi}^{\pi} \int_0^{\tau_{\max}} S(\theta, \tau') \mathbf{a}(\theta) \mathbf{b}^T(\phi) g(\tau - \tau') d\tau' d\theta$$

- Because of flat fading, it becomes,

$$\mathbf{H}(\tau) = \left( \int_{-\pi}^{\pi} \int_0^{\tau_{\max}} S(\theta, \tau') \mathbf{a}(\theta) \mathbf{b}^T(\phi) d\tau' d\theta \right) g(\tau) = \mathbf{H} g(\tau)$$

$\mathbf{a}$  and  $\mathbf{b}$  are transmit and receive array factor vectors respectively.  $S$  is the complex gain that is dependant on direction and delay.  $g(t)$  is the transmit and receive pulse shaping impulse response

- With suitable choices of array geometry and antenna element patterns,  $\mathbf{H}(\tau) = \mathbf{H}$  which is an  $M_R \times M_T$  matrix with complex Gaussian i. i. d random variables
- Accurate for NLOS rich-scattering environments, with sufficient antenna spacing at transmitter and receiver with all elements identically polarized



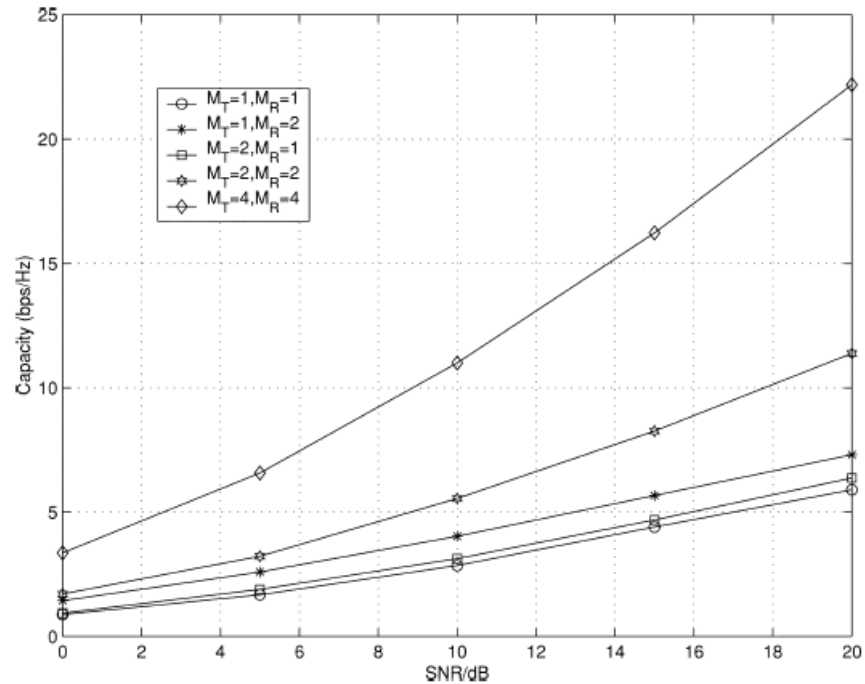
# Capacity of MIMO Channels

- $y = Hs + n$
- Let the transmitted vector  $s$  be a random vector to be very general and  $n$  is normalized noise. Let the total transmitted power available per symbol period be  $P$ . Then,
- $C = \log_2 (I_M + HQHH) \text{ b/s/Hz}$
- where  $Q = E\{ss^H\}$  and  $\text{trace}(Q) < P$  according to our power constraint
- Consider specific case when we have users transmitting at equal power over the channel and the users are uncorrelated (no feedback available). Then,
- $CEP = \log_2 [I_M + (P/MT)HHH] \text{ b/s/Hz}$
- Telatar showed that this is the optimal choice for blind transmission
- Foschini and Telatar both demonstrated that as  $MT$  and  $MR$  grow,
- $CEP = \min(MT, MR) \log_2 (P/MT) + \text{constant} \text{ b/s/Hz}$
- Note: When feedback is available, the Waterfilling solution yields maximum capacity but converges to equal power capacity at high SNRs



# Capacity (contd)

- The capacity expression presented was over one realization of the channel. Capacity is a random variable and has to be averaged over infinite realizations to obtain the true ergodic capacity. Outage capacity is another metric that is used to capture this



- So MIMO promises enormous rates theoretically! Can we exploit this practically?**



- MIMO Systems can provide two types of gain

## Spatial Multiplexing Gain



- Maximize transmission rate (optimistic approach)
- Use rich scattering/fading to your advantage

## Diversity Gain



- Minimize  $P_e$  (conservative approach)
- Go for Reliability / QoS etc
- Combat fading

- If only I could have both! As expected, there is a tradeoff
- System designs are based on trying to achieve either goal or a little of both



- Each pair of transmit-receive antennas provides a signal path from transmitter to receiver. By sending the SAME information through different paths, multiple independently-faded replicas of the data symbol can be obtained at the receiver end. Hence, more reliable reception is achieved
- A diversity gain  $d$  implies that in the high SNR region, my  $P_e$  decays at a rate of  $1/\text{SNR}^d$  as opposed to  $1/\text{SNR}$  for a SISO system
- The maximal diversity gain  $d_{\max}$  is the total number of independent signal paths that exist between the transmitter and receiver
- For an  $(M_R, M_T)$  system, the total number of signal paths is  $M_R M_T$
- 
- $1 \leq d \leq d_{\max} = M_R M_T$
- The higher my diversity gain, the lower my  $P_e$



$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \rightarrow \mathbf{y}' = \mathbf{D}\mathbf{s}' + \mathbf{n}' \text{ (through SVD on } \mathbf{H}\text{)}$$

where  $\mathbf{D}$  is a diagonal matrix that contains the eigenvalues of  $\mathbf{H}\mathbf{H}^H$

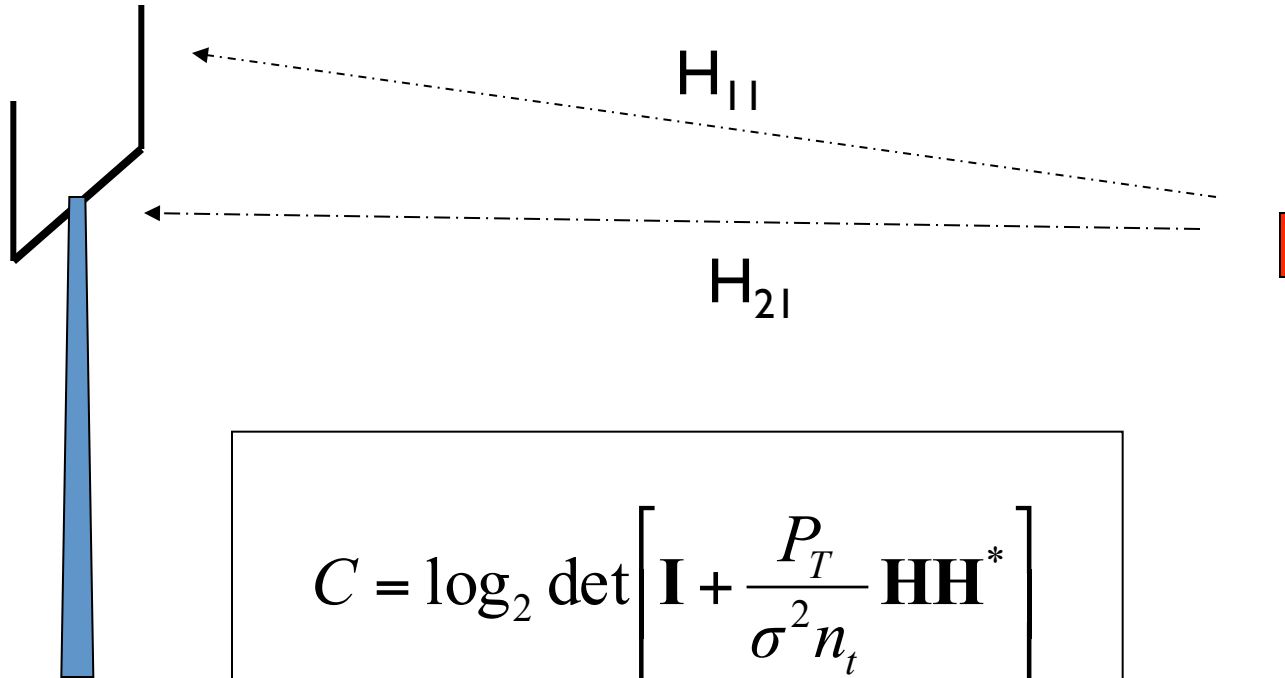
- Viewing the MIMO received vector in a different but equivalent way,  
$$C_{EP} = \log_2 [\mathbf{I}_M + (\mathbf{P}/M_T)\mathbf{D}\mathbf{D}^H] = \log_2 [1 + (\mathbf{P}/M_T)\lambda_i] \text{ b/s/Hz}$$
- Equivalent form tells us that an  $(M_T, M_R)$  MIMO channel opens up  $m = \min(M_T, M_R)$  independent SISO channels between the transmitter and the receiver
- So, intuitively, I can send a maximum of  $m$  different information symbols over the channel at any given time



## Initial Assumptions

- Flat fading channel ( $B_{\text{coh}} \gg 1/T_{\text{symb}}$ )
- Slowly fading channel ( $T_{\text{coh}} \gg T_{\text{symb}}$ )
- $n_r$  receive and  $n_t$  transmit antennas
- Noise limited system (no CCI)
- Receiver estimates the channel perfectly
- We consider space diversity only

# Receive Diversity



$$C = \log_2 \det \left[ \mathbf{I} + \frac{P_T}{\sigma^2 n_t} \mathbf{H} \mathbf{H}^* \right]$$

$$= \log_2 [1 + (P_T/\sigma^2) \cdot |\mathbf{H}|^2] \quad [\text{bit}/(\text{Hz} \cdot \text{s})]$$

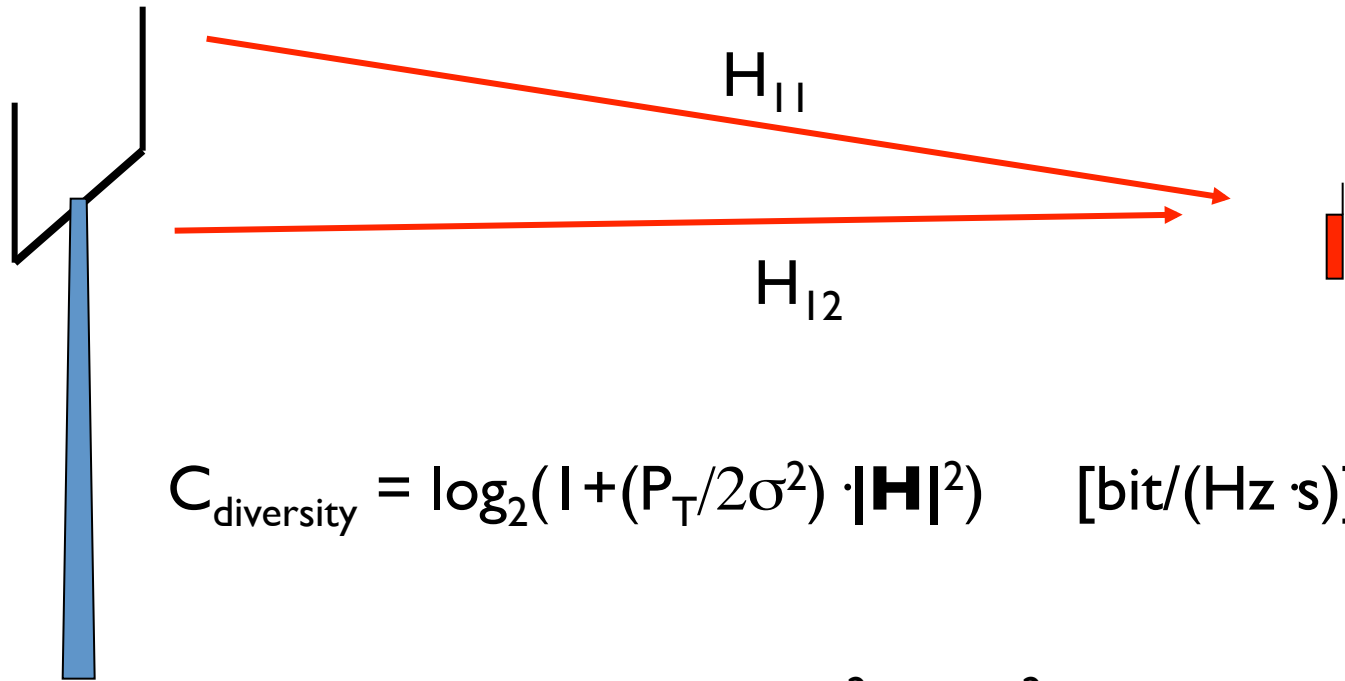


Capacity increases logarithmically  
with number of receive antennas...

$$\mathbf{H} = [H_{11} \ H_{21}]$$



# Transmit Diversity

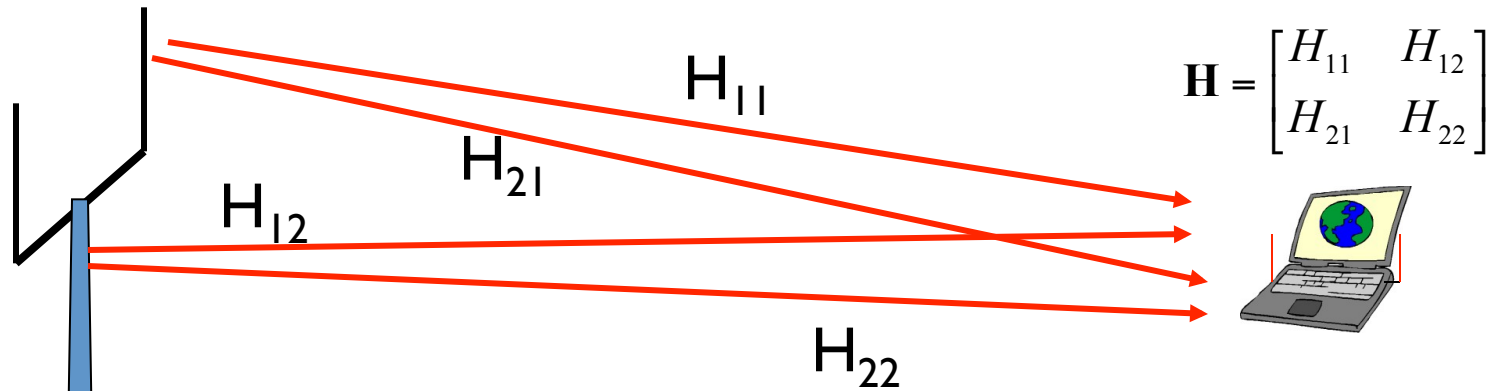


$$C_{\text{diversity}} = \log_2(1 + (P_T/2\sigma^2) \cdot |\mathbf{H}|^2) \quad [\text{bit}/(\text{Hz} \cdot \text{s})]$$

$$C_{\text{beamforming}} = \log_2(1 + (P_T/\sigma^2) \cdot |\mathbf{H}|^2) \quad [\text{bit}/(\text{Hz} \cdot \text{s})]$$



- 3 dB SNR increase if transmitter knows  $\mathbf{H}$
- Capacity increases logarithmically with  $n_t$

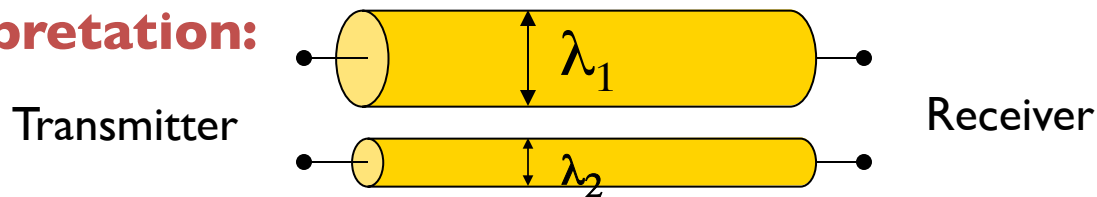


$$C_{\text{diversity}} = \log_2 \det[\mathbf{I} + (P_T/2\sigma^2) \cdot \mathbf{H}\mathbf{H}^\dagger] =$$

Where the  $\lambda_i$  are the eigenvalues to  $\mathbf{H}\mathbf{H}^\dagger$

$$= \log_2 \left[ 1 + \frac{P_T}{2\sigma^2} \lambda_1 \right] + \log_2 \left[ 1 + \frac{P_T}{2\sigma^2} \lambda_2 \right]$$

**Interpretation:**



$m = \min(n_r, n_t)$  parallel channels,  
equal power allocated to each "pipe"



# MIMO capacity in general

H unknown at TX

$$C = \log_2 \det \left[ I + \frac{P_T}{\sigma^2 n_t} HH^* \right] =$$

$$= \sum_{i=1}^m \log_2 \left[ 1 + \frac{P_T}{\sigma^2 n_t} \lambda_i \right]$$

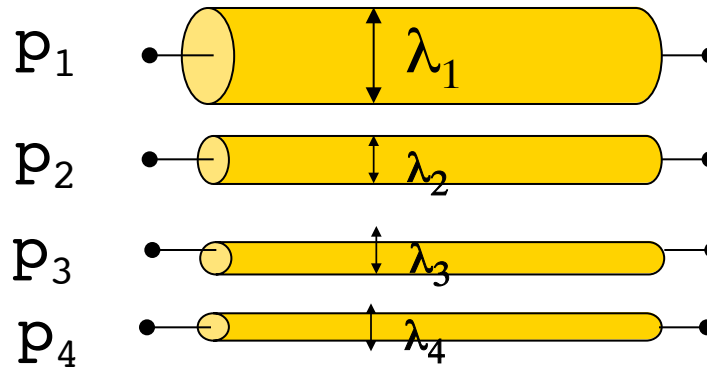
$$m = \min(n_r, n_t)$$

H known at TX

$$C = \sum_{i=1}^m \log_2 \left[ 1 + \frac{p_i \lambda_i}{\sigma^2} \right]$$

Where the power distribution over "pipes" are given by a water filling solution

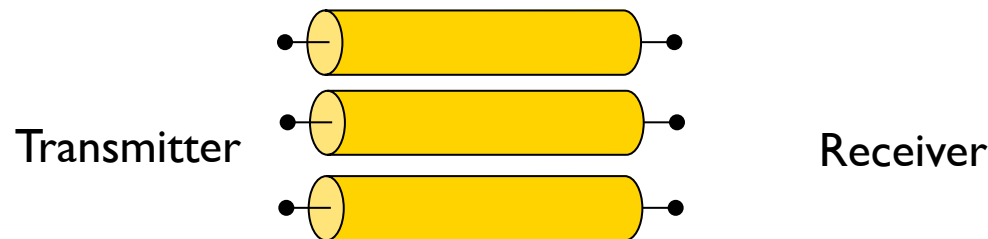
$$P_T = \sum_{i=1}^m p_i = \sum_{i=1}^m \left( \nu - \frac{1}{\lambda_i} \right)^+$$

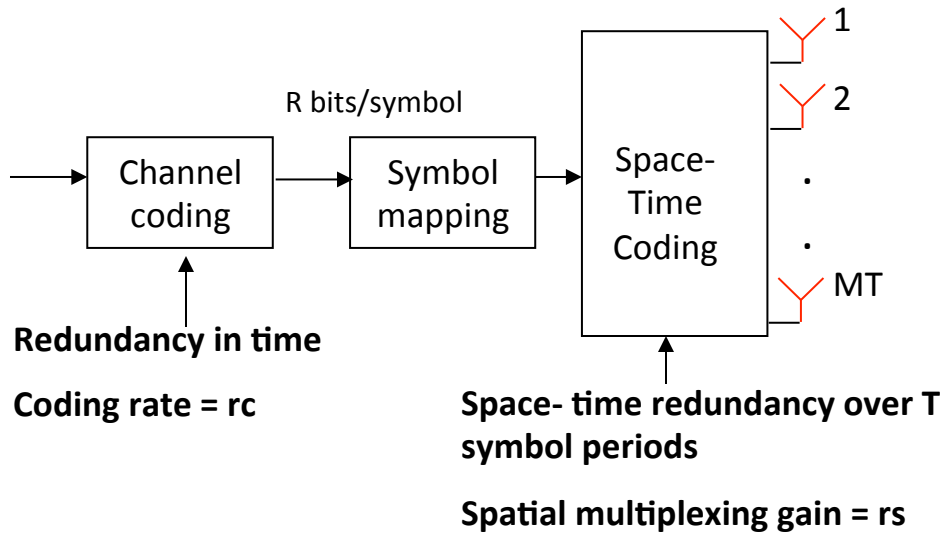


**Orthogonal channels**  $\mathbf{H}\mathbf{H}^\dagger = \mathbf{I}$ ,  $\lambda_1 = \lambda_2 = \dots = \lambda_m = 1$

$$C_{\text{diversity}} = \sum_{i=1}^m \log_2 \left[ 1 + \frac{P_T}{\sigma^2 n_t} \lambda_i \right] = \min(n_t, n_r) \cdot \log_2 (1 + P_T / \sigma^2 n_t)$$

- ➔
- Capacity increases linearly with  $\min(n_r, n_t)$
  - An equal amount of power  $P_T/n_t$  is allocated to each "pipe"





**rs : number of different symbols N transmitted in T symbol periods**

$$rs = N/T$$

Non-redundant portion of symbols

$$\text{Spectral efficiency} = \frac{(R \cdot rc \text{ info bits/symbol})(rs)(R_s \text{ symbols/sec})}{w}$$

$w$

$$= Rrcrs \text{ bits/s/Hz assuming } R_s = w$$

rs is the parameter that we are concerned about:  $0 \leq rs \leq MT$

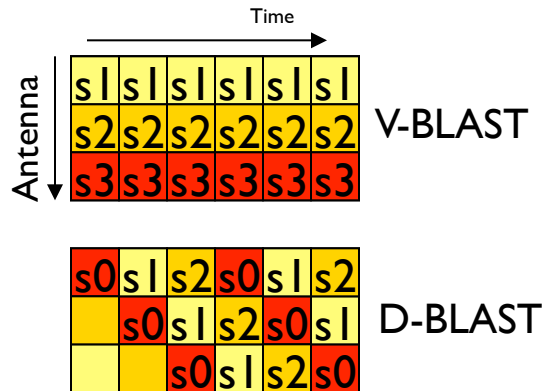
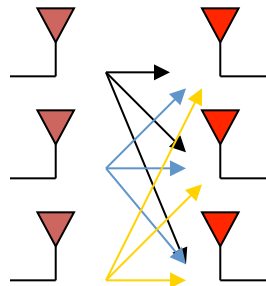
**\*\* If  $rs = MT$ , we are in spatial multiplexing mode (max transmission rate)**

**\*\*If  $rs \leq 1$ , we are in diversity mode**





Bell Labs Layered Space Time Architecture

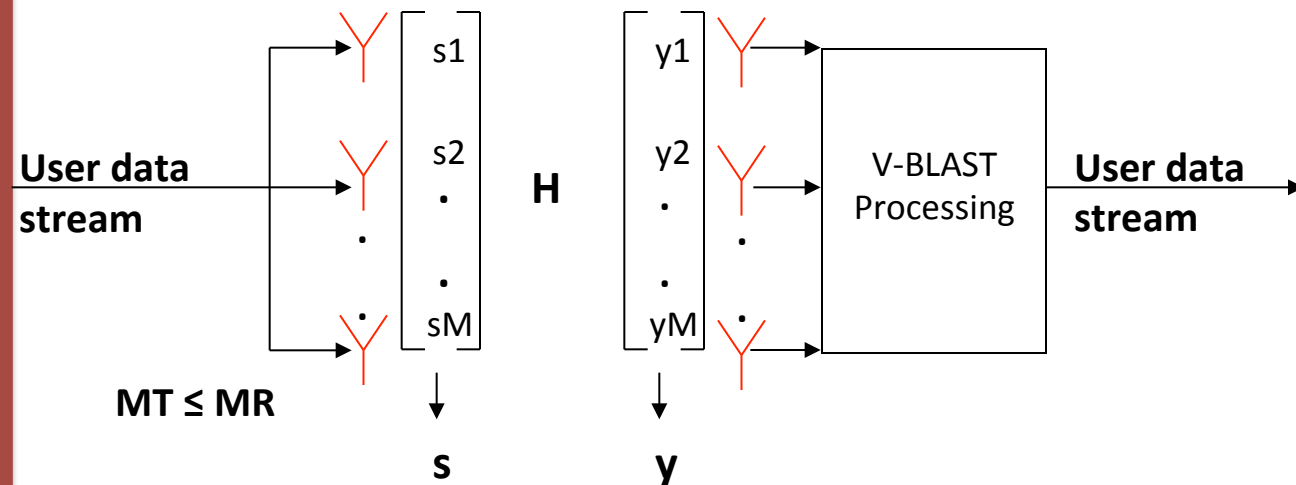


- $n_r \geq n_t$  required
- Symbol by symbol detection. Using nulling and symbol cancellation
- V-BLAST implemented -98 by Bell Labs (40 bps/Hz)
- If one "pipe" is bad in BLAST we get errors ...



# V-BLAST (Vertical) – Spatial Multiplexing

- This is the only architecture that goes all out for maximum rate. Hope the channel helps me out by ‘splitting’ my info streams!



- Split data into MT streams → maps to symbols → send
- Assume receiver knows H
- Uses old technique of ordered successive cancellation to recover signals
- Sensitive to estimation errors in H
- rs = MT because in one symbol period, you are sending MT different symbols

initialization:

$$i \leftarrow 1$$

$$\mathbf{G}_1 = \mathbf{H}^+$$

$$k_1 = \underset{j}{\operatorname{argmin}} \|(\mathbf{G}_1)_j\|^2$$

recursion:

$$\mathbf{w}_{k_i} = (\mathbf{G}_i)_{k_i}$$

$$y_{k_i} = \mathbf{w}_{k_i}^T \mathbf{r}_i$$

$$\hat{a}_{k_i} = Q(y_{k_i})$$

$$\mathbf{r}_{i+1} = \mathbf{r}_i - \hat{a}_{k_i} (\mathbf{H})_{k_i}$$

$$\mathbf{G}_{i+1} = \mathbf{H}_{\bar{k}_i}^+$$

$$k_{i+1} = \underset{j \in \{k_1, \dots, k_i\}}{\operatorname{argmin}} \|(\mathbf{G}_{i+1})_j\|^2$$

$$i \leftarrow i + 1$$



# D-BLAST – (Diagonal)

- In D-BLAST, the input data stream is divided into sub streams which
- are coded, each of which is transmitted on different antennas time
- slots in a diagonal fashion
- For example, in a (2,2) system

$$\begin{bmatrix} 0 & \mathbf{x}_1^{(1)} & \mathbf{x}_1^{(2)} & \dots \\ \mathbf{x}_2^{(1)} & \mathbf{x}_2^{(2)} & \mathbf{x}_2^{(3)} & \dots \end{bmatrix}$$

$MT \leq MR$

- receiver first estimates  $\mathbf{x}_2^{(1)}$  and then estimates  $\mathbf{x}_1^{(1)}$  by treating  $\mathbf{x}_2^{(1)}$  as interference and nulling it out
- The estimates of  $\mathbf{x}_2^{(1)}$  and  $\mathbf{x}_1^{(1)}$  are fed to a joint decoder to decode the first substream

- After decoding the first substream, the receiver cancels the contribution of this substream from the received signals and starts to decode the next substream, etc.
- Here, an overhead is required to start the detection process; corresponding to the 0 symbol in the above example
- Receiver complexity high

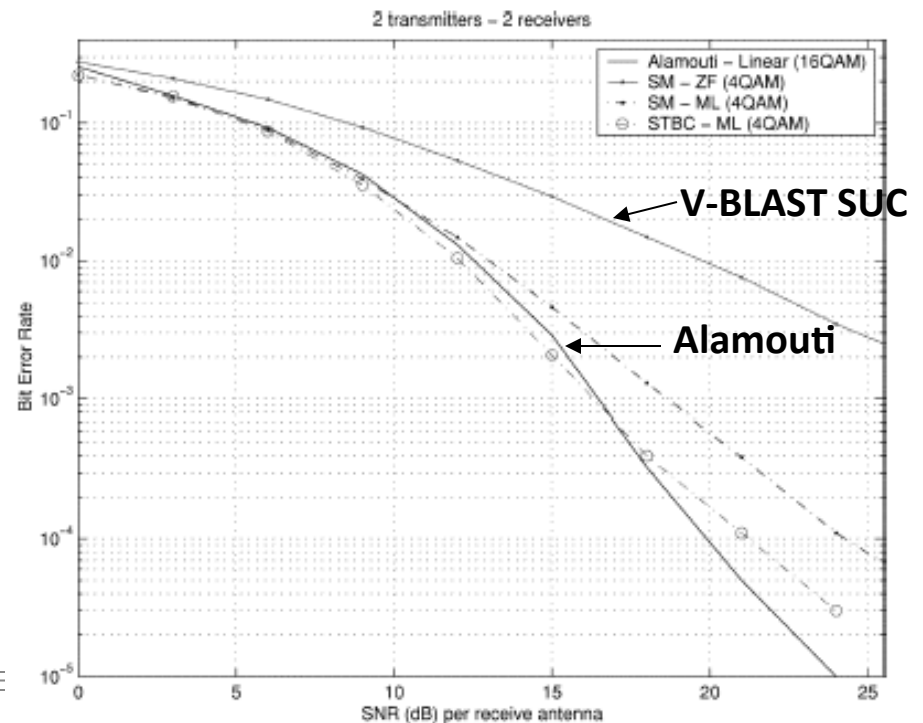
# Alamouti's Scheme - Diversity

- Transmission/reception scheme easy to implement
- Space diversity because of antenna transmission. Time diversity because of transmission over 2 symbol periods
- Consider (2, MR) system

$$\begin{bmatrix} \mathbf{x}_1 & -\mathbf{x}_2^\dagger \\ \mathbf{x}_2 & \mathbf{x}_1^\dagger \end{bmatrix}$$

- Receiver uses combining and ML detection
- $r_s = 1$

- If you are working with a (2,2) system, stick with Alamouti!
- Widely used scheme: CDMA 2000, WCDMA and IEEE 802.16-2004 OFDM-256





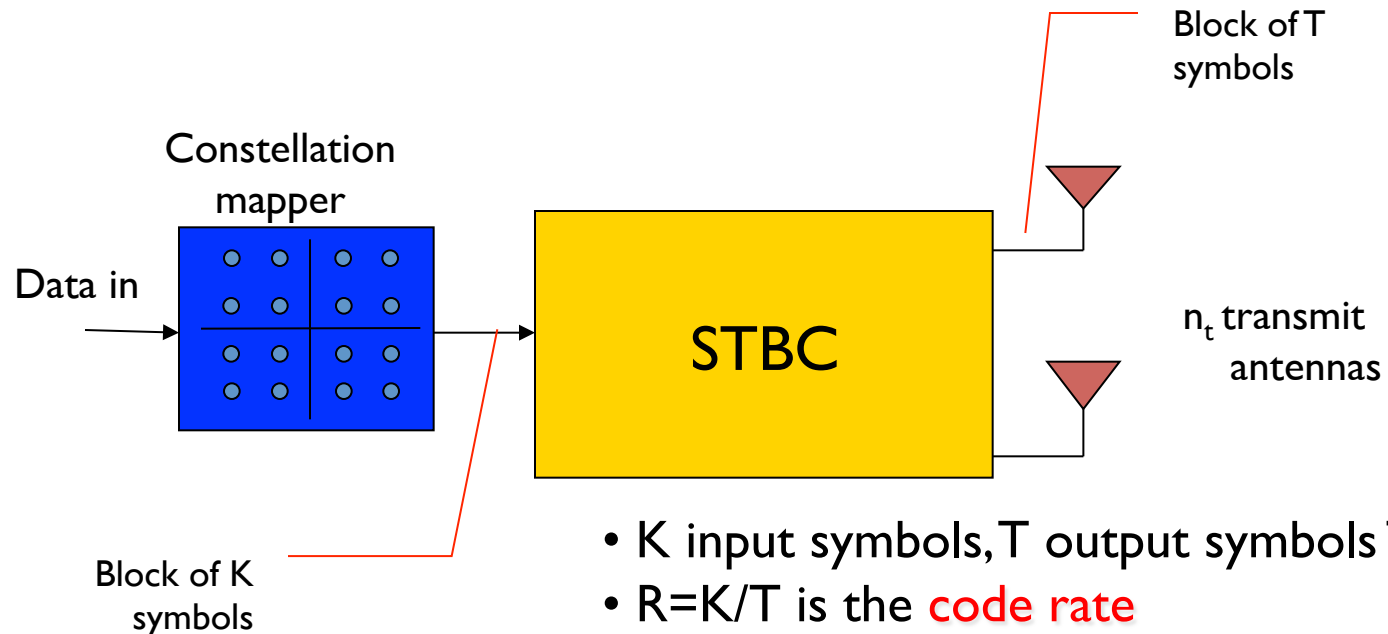
Scheme	Spectral Efficiency	$P_e$	Implementation Complexity
V-BLAST	HIGH	HIGH	LOW
D-BLAST	MODERATE	MODERATE	HIGH
ALAMOUTI	LOW	LOW	LOW



- Use parallel channel to obtain **diversity** not spectral efficiency as in BLAST
- Space-Time **trellis** codes : coding **and** diversity gain (require Viterbi detector)
- Space-Time **block** codes : diversity gain (use outer code to get coding gain)
- $n_r = 1$  is possible
- Properly designed codes achieve diversity of  $n_r n_t$

\*{V.Tarokh, N.Seshadri, A.R.Calderbank  
Space-time codes for high data rate wireless communication:  
Performance Criterion and Code Construction  
, IEEE Trans. On Information Theory March 1998 }

# Orthogonal Space-time Block Codes

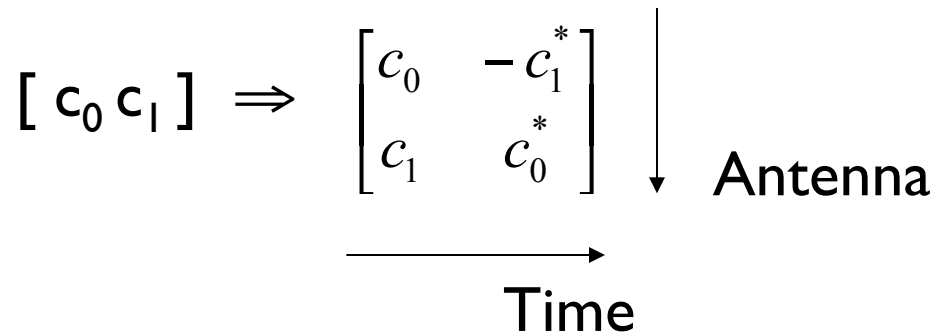


- K input symbols, T output symbols  $T \geq K$
- $R=K/T$  is the **code rate**
- If  $R=1$  the STBC has **full rate**
- If  $T= n_t$  the code has **minimum delay**
- Detector is **linear** !!!

\*{V.Tarokh, H.Jafarkhani, A.R.Calderbank  
Space-time block codes from orthogonal designs,  
IEEE Trans. On Information Theory June 1999}

# STBC for 2 Transmit Antennas

Full rate and  
minimum delay



Assume 1 RX antenna:

Received signal at time 0

$$r_0 = h_1 c_0 + h_2 c_1 + n_0$$

Received signal at time 1

$$r_1 = -h_1 c_1^* + h_2 c_0^* + n_1$$



$$\longrightarrow \mathbf{r} = \bar{\mathbf{H}}\mathbf{c} + \mathbf{n}$$

$$\mathbf{r} = \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix}, \quad \bar{\mathbf{H}} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

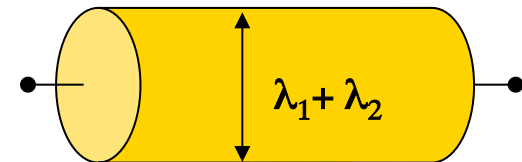
$$\tilde{\mathbf{r}} = \bar{\mathbf{H}}^* \mathbf{r} = \underbrace{\bar{\mathbf{H}}^* \bar{\mathbf{H}}}_{\text{Diagonal matrix due to orthogonality}} \mathbf{c} + \bar{\mathbf{H}}^* \mathbf{n} = \|\mathbf{H}\|_F^2 \mathbf{c} + \tilde{\mathbf{n}}$$

Diagonal matrix due to orthogonality

The MIMO/ MISO system is in fact transformed to an equivalent SISO system with SNR

$$\text{SNR}_{\text{eq}} = \|\mathbf{H}\|_F^2 \text{SNR}/n_t$$

$$\|\mathbf{H}\|_F^2 = \lambda_1 + \lambda_2$$



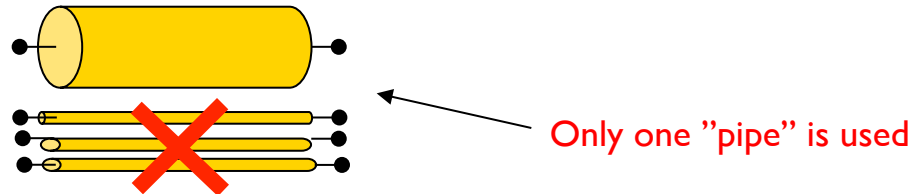
# MIMO With Beamforming



Requires that channel  $\mathbf{H}$  is known at the transmitter  
Is the capacity-optimal transmission strategy if

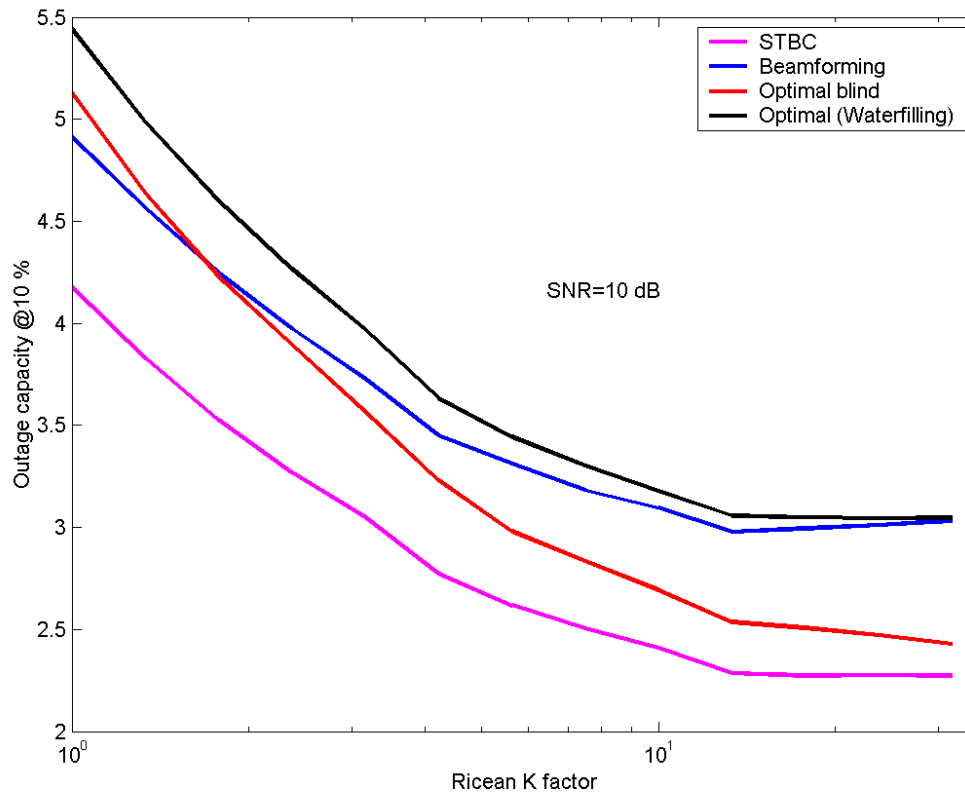
$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \geq SNR$$

Which is often true for line of sight (LOS) channels



$$C_{\text{beamforming}} = \log_2(1 + SNR \cdot \lambda_1) \quad [\text{bit}/(\text{Hz} \cdot \text{s})]$$

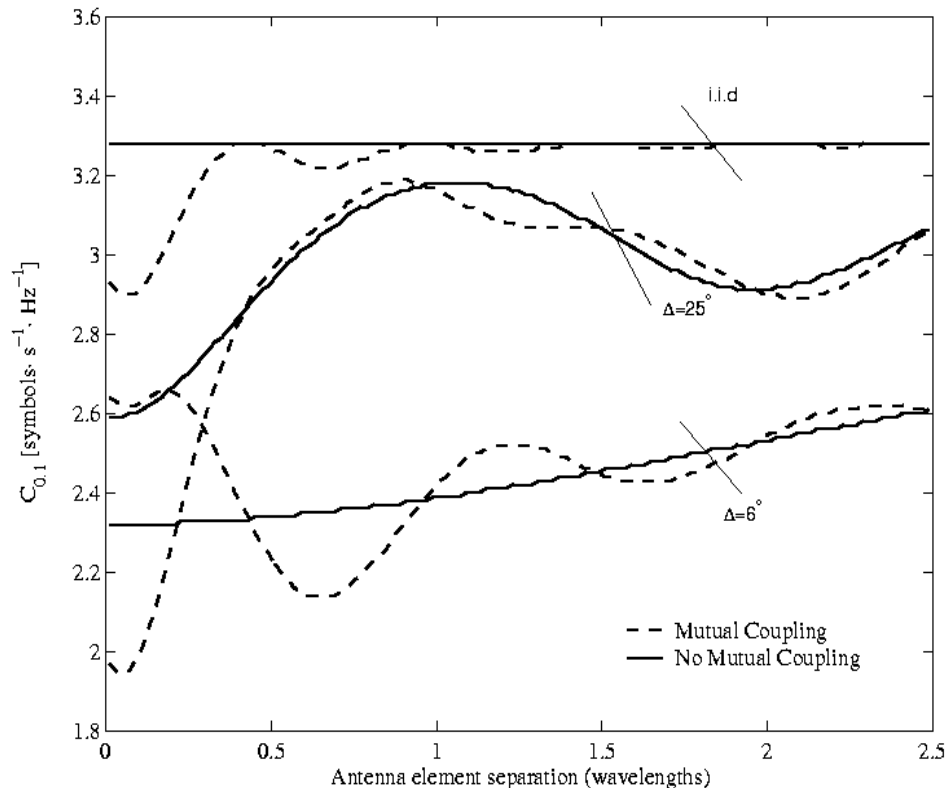
2 \* 2 system. With specular component (Ricean fading)



} One dominating eigenvalue. BF puts all energy into that "pipe"



# Correlated channels/mutual coupling



When angle spread ( $\Delta$ ) is small, we have a dominating eigenvalue. The mutual coupling actually improves the performance of the STBC by making the eigenvalues "more equal" in magnitude.

(3GPP Release '99 with 2 TX antennas)

- **2 modes**
  - **Open loop (STTD)** ↗ **Open loop mode is exactly the 2 antenna STBC**
  - **Closed loop (1 bit / slot feedback)**
    - **Submode 1 (1 phase bit)**
    - **Submode 2 (3 phase bits / 1 gain bit)**

$$\begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix}$$



The feedback bits (1500 Hz) determines the beamformer weights

**Submode 1** Equal power and bit chooses phase between  $\{0, 180\}$  /  $\{90/270\}$

**Submode 2** Bit one chooses power division  $\{0.8, 0.2\}$  /  $\{0.2, 0.8\}$  and 3 bits chooses phase in an 8-PSK constellation



- **Channel capacity increases linearly**  
**with  $\min(n_r, n_t)$**
- **STBC is in the 3GPP WCDMA proposal**



- Adapted from notes of **HARISH GANAPATHY and Mattias Wennström**
- “Layered Space-Time Architecture for Wireless Communication in a Fading Environment When using Multi-Element Antennas”, G.J.Foschini, Bell Labs Tech Journal, 1996
- “An Overview of MIMO Communications – A Key to Gigabit Wireless”, A.J Paulraj, Gore, Nabar and Bolcskei, IEEE Trans Comm, 2003
- “Improving Fairness and Throughput of Ad Hoc Networks Using Multiple Antennas”, Park, Choi and Nettles, submitted Mobicom 2004
- “From Theory to Practice: An Overview of MIMO Space-Time Coded Wireless Systems”, Gesbert et al., IEEE Sel Comm, 2003
- “On Limits of Wireless Communications in a Fading Environment”, Foschini and Gans, Wireless Personal Comm, 1998
- “A Simple Transmit Diversity Technique for Wireless Communications”, Alamouti, IEEE Sel Comm, 1998
- “Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels”, Zheng and Tse, IEEE Trans Info Theory, 2003
- “V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel”, Wolniansky, Foschini, Golden and Valenzuela, Electronic Letters, 1999
- “MIMO-OFDM Systems for High Data Rate Wireless Networks”, Whu